Theory Of Machine Design

Unit-1 Mechanism

Theory of Machine + machine Design
(30%) (70%)

→ **Theory of machine:**
It deals the theoretical aspect of machine. It analysis the strength of various parts and also predicts the geometrical arrangement of the element.

We mainly deal with:

→ kinematics

→ kinematic Link or element: Different parts of machine that has relative motion to each other is known as kinematic link or element

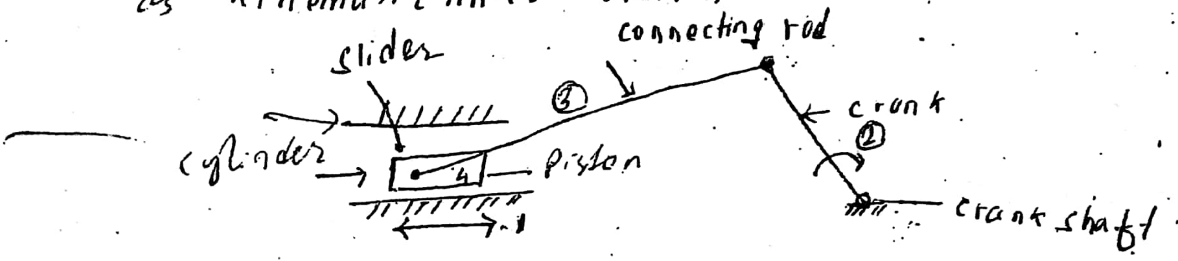


Fig: Reciprocating pump

From above figure:
Link are:

Pair ⟨ cylinder, piston, piston rod

Pair ⟨ connecting rod

Pair ⟨ crank

Pair ⟨ crankshaf

A link or element need not to be a rigid body, but it must be a resistant body. A body is said to be a resistant body if it is capable of transmitting the required forces with negligible deformation. Thus, a link should have the following two characteristics:

(1) It should have relative motion.
(2) It must be a resistant body.

→ Types of Links:

① Rigid link: A rigid link is one which does not undergo any deformation while transmitting motion.

② Flexible link: A flexible link is one which is partly deformed in a manner not to affect the transmission of motion.

③ Fluid link: A fluid link is one which is formed by having a fluid in a receptacle and motion is transmitted through the fluid pressure.

⇒ Kinematic pairs:

The kinematic links are connected in such way that the relative motion between the element is constrained. Such type of points is called kinematic pairs.

Types of kinematic pairs :-

(1) On the basis of relative motion between the elements

(a) Sliding pair (prismatic pair) :- when the two element of a pair are connected in such a way that one can only slide relative to the other, the pair is known as sliding pair. ex: piston and cylinder, cross-head and guides.

(b) Turning pair :- — turn or revolve about a fixed axis of another link.

(c) Rolling pair :- that rolls over another fixed link. eg:- Ball and roller bearing.

(d) Screw pair :- - one element can turn about the other by screw threads. eg: nut and bolt.

(e) Spherical pair :- one element turns or swivels about the other fixed element, the pair formed is called a spherical pair.

(2) on the basis of of nature of contact between the element

(a) Lower pair — surface contact

(b) Higher pair - pt or line contact.

⇒ kinematic chain :

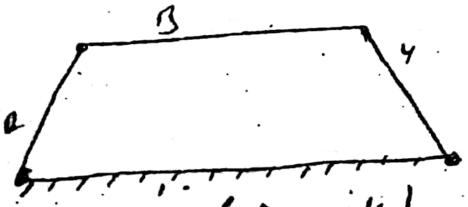
When the kinematic links are connected by a numbers of pair such that the first links is connected to last links to transmit constrained motion, then such type of chain is called kinematic chain.

For ex: the crankshaft of an engine form a kinematic pair of ~~an engine~~ with the bearing which are fixed in a pair ; the connecting rod with the crank forms a second ~~kinect~~ kinematic pair, the piston with the connecting rod form a third pair and the piston with the cylinder forms a fourth pair. The total combination of these links is a kinematic chain.

Mechanism :

When any one links of the kinematic chain is fixed. Then it is called mechanism.

For complete mechanism it need at least four links.



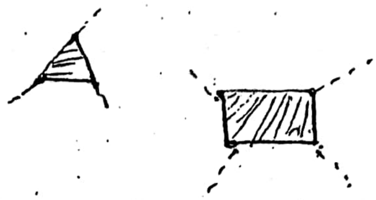
→ Simple mechanism (n = 4)

→ compound " (no of links > 4)

In such case, the various links or element have to be designed to withstand the force (both static and kinetic) safely.

Link

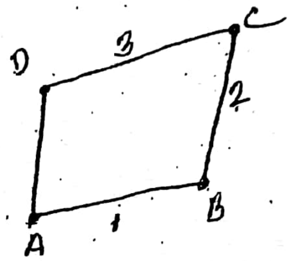
- binary links
- Ternary links
- quaternary "



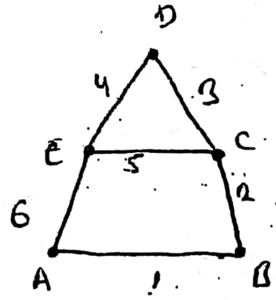
pair :- (joint)

→ binary pair :- when two links are joined at the same connection the joint is known as binary joint. For example, a chain as shown in ; has four link and four binary joint A, B C and D.

kinematic chain with binary joints.



→ ternary joint :- when three link are joined at the same connection, the joint is known as ternary joint.



→ Quaternary joint :- when four link are joined at the same connection, the joint is called a quaternary joint.

⇒ Grashoff's Law:

If l and s be the length of longest and shortest length and p and Q be length of the remaining two elements of the planner mechanism, Then

$$(l+s) \le (P+Q)$$

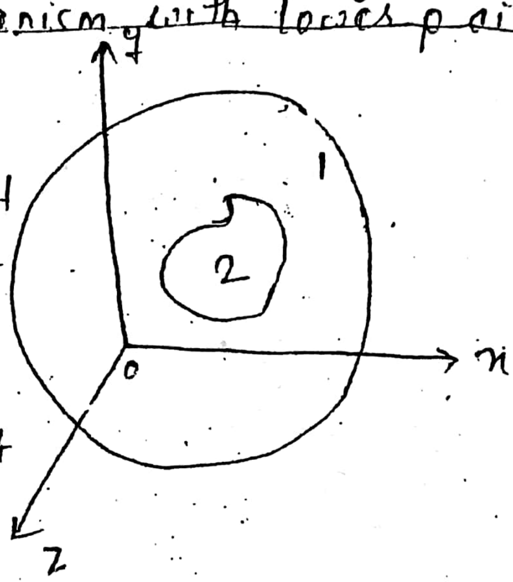
The mechanism which satisfy above relation is called Grashoff's linkage.

V.V.I.

Degree of freedom of mechanis:

Grueblers criterion for constrain-t motion of planar mechanism with lower pair:

Consider element 2 having relative motion to the element 1 and O_{xyz} be the an element of 1.



The maximum degree of freedom of link on a plane is 3

Consider a kinematic chain with n elements connected by j no of turning pair having, total no. of degree of freedom F.

For a mechanism any one (1) link of the kinematic chain should be fixed.

The total degree of freedom = $3(n-1)$

For J no of turning pairs,

Total degree of freedom of the system

$$F = 3(n-1) - 2j \quad \text{——(i)}$$

If the kinematic chain are made up of different types of links. Then,

$$j = \frac{1}{2}(2n_2 + 3n_3 + \cdots\cdots + in_i)$$

where,

n_2 = No. of binary links

n_3 = No. of ternary links

N_i = no. of links with i hinge joint.

For, eq (i)

If $F = 1$

$$3n - 2j = 4$$

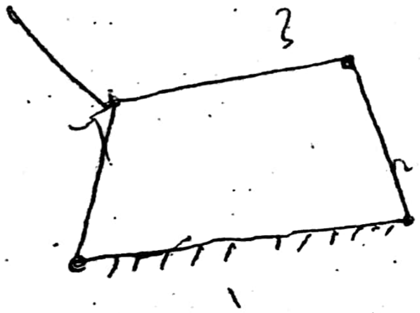
If the chain has h no of higher pairs.

$$F = 3(n-1) - 2j - h$$

If $F = 0$ determine structure

If $F \geq 1$ mechanism

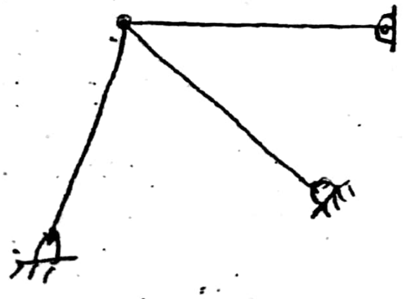
If $F < 1$ structure



No of element (n) = 4.

(J) = 4

$$F = 3(n-1) - 2j$$
$$= 3(4-1) - 2\times 4$$
$$= 9 - 8 = 1$$



$n = 3$

$j = 2$

$h = 1$

$n = 4$

$j = 3+2$

$F = 9 - 10 = -1$

$$F = 3(n-1) - 2j - h$$
$$= 3(3-1) - 2\times 2 - 1$$
$$= 6 - 4 - 1$$
$$= 1$$

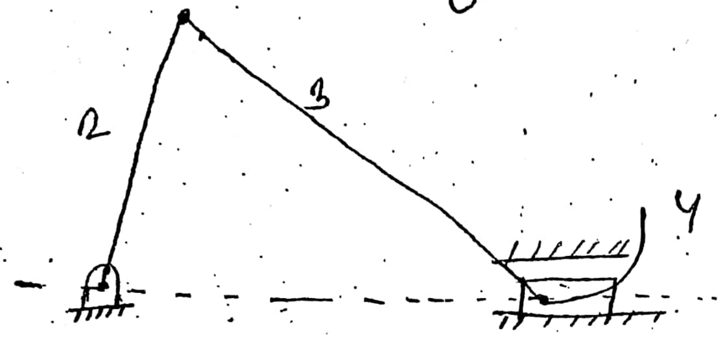
Inversion of Mechanism:

The process of fixing of different links of the same kinematic chain to obtain different mechanism is called inversion of mechanism.

~~It~~

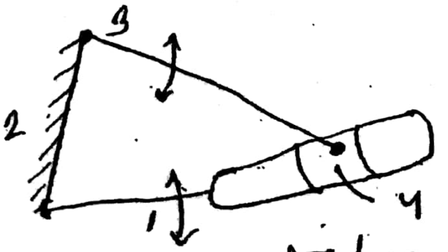
eg:- Single slider crank mechanism.

① If 1st link (crank) is fixed



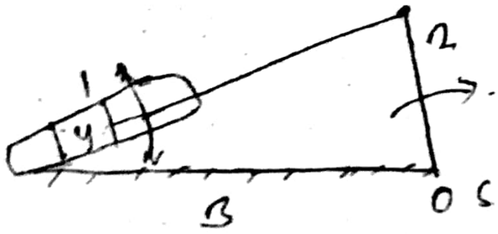
Single slider crank chain is a four-link mechanism. We know that by fixing, in turn, different links in a kinematic chain, an inversion is obtained and we can obtain as many mechanisms as the links in a kinematic chain. It is thus obvious that four inversion of a single slider crank chain are possible. These inversion are found in the following mechanism.

② If 2nd link is fixed



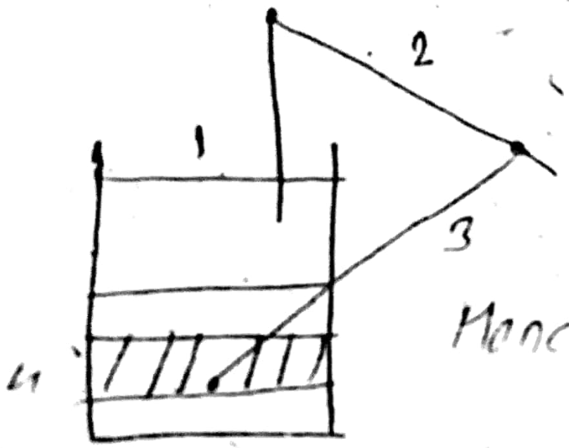
rotary engine

If link 3 is fixed



oscillating engine

(iii). If link 4 is fixed



Hand pump

Unit - 2

Analysis of Mechanism :

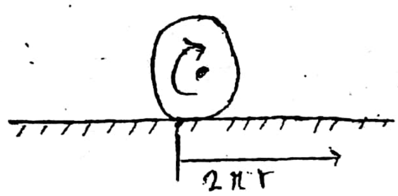
The velocity at a point on a link of mechanism can be determined by

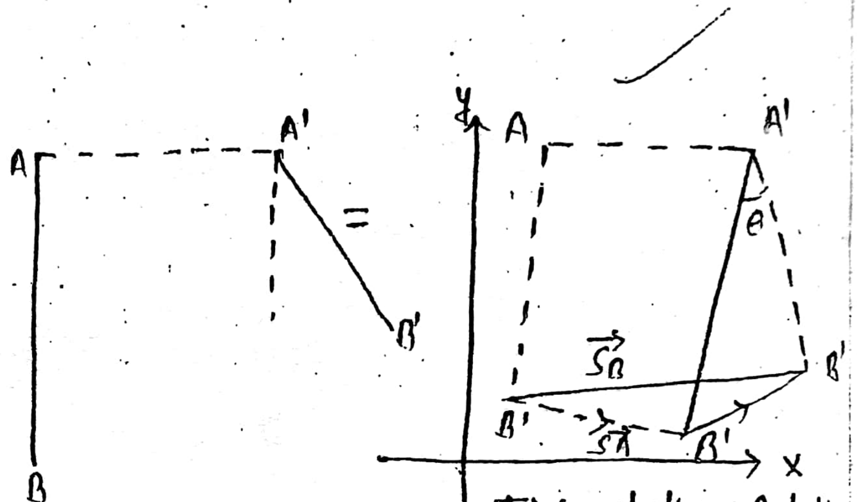
(a) Relative method → Graphical method.

(b) Instantaneous centre method (I c) method

Plane motion : Combination of translation and rotation.

(a) anaylitical method :





Translation + Rotation

Note : $\vec{a_c} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$

$$\vec{S_B} = \vec{S_A} + \vec{S}_{B/A}$$

$$\left\{ \begin{array}{l} \vec{V_B} = \vec{V_A} + \vec{V}_{B/A} \\ \vec{a_B} = \vec{a_A} + \vec{a}_{B/A} \end{array} \right.$$

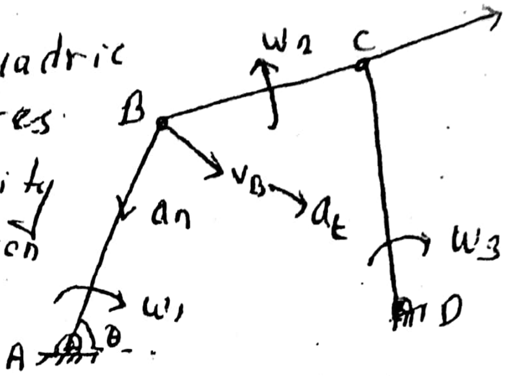
$$\vec{V_B} = \vec{V_A} + \vec{\omega} \times \vec{BA}$$

$$\vec{a_B} = \vec{a_A} + \vec{a}_{B/A}$$

$$= \vec{a_a} + \vec{\alpha} \times \vec{BA} + \vec{\omega} \times (\vec{\omega} \times \vec{BA})$$

→ <u>velocity and accleration at a point on link of quadric cycle mechanism :</u>

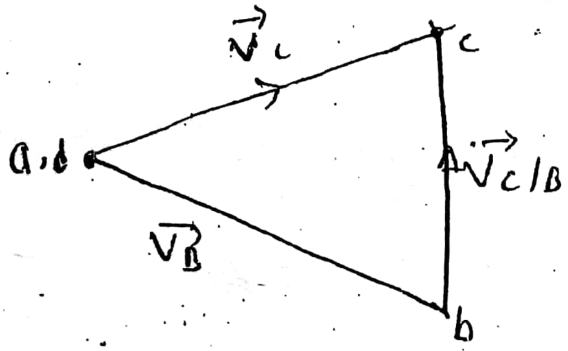
Consider a crank AB of quadric cycle mechanism rotates with constant angular velocity ω_1 in clock-wise direction for a given position as shown in figure.



(i) ~~Fro~~ For velocity :

$V_B = W_1 \cdot AB$

To draw velocity diagram.



ab ⊥ AB
bc ⊥ BC
dc ⊥ CD

From velocity vector diagram

$ab = |\vec{V_B}| = \ldots \, m/s$

$ac = |\vec{V_c}| = \ldots \, m/s$

$bc = |\vec{V_{c/B}}| = \ldots \, m/s$

For link CD

$$V_c = W_3 \cdot CD$$

$$\omega_3 = \frac{V_c}{CD} = \text{-----} \text{ radain } (2)$$

For link BC :

$$V_{B/C} = \omega_2 \cdot BC$$

$$\omega_2 = \frac{V_{B/C}}{BC} = \text{----} \text{ radian } (4)$$

(ii) Accleration:

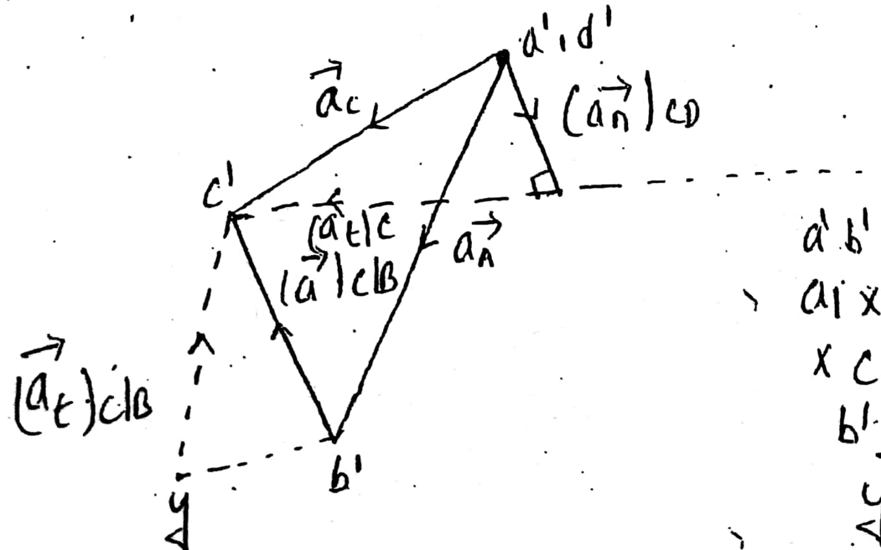
To draw accleration vector diagram

For link AB.

$$a = a_n$$

$$= \omega_1^2 \cdot AB = \text{----} \; m/s^2$$

$$\vec{a_A} = (\vec{a_n})_{AB}$$



$a'b' \parallel AB$

$a'x \parallel CD$

$xc' \perp CD$

$b'y \parallel BC$

$yc' \perp BC$

For link CD

$$\vec{ac} = (\vec{a_t})_{CD} + (\vec{a_n})_{CD}$$

$$a_t = \alpha_3 \cdot CD = xc' \Rightarrow \alpha_3 = \frac{xc'}{CD} = \text{-----} \; rad/se^2 (4)$$

$a_n = \omega_3'^2 \cdot CD$

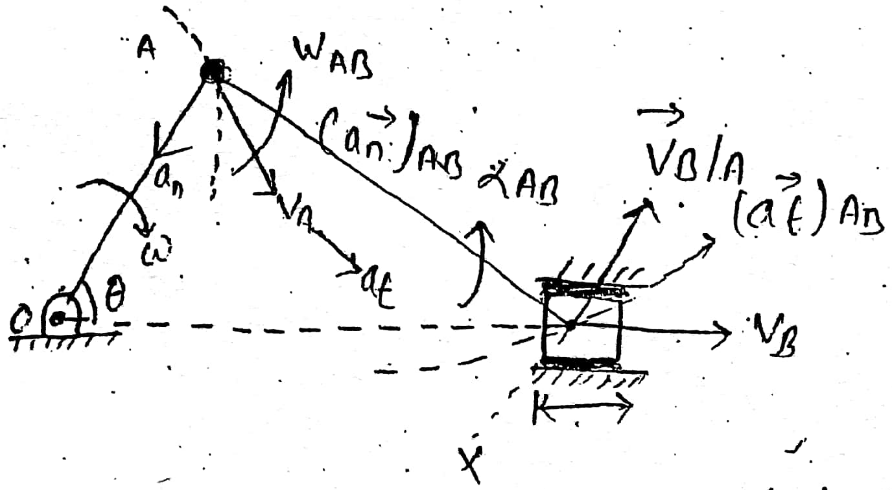
For link BC,

$a_{C/B} = (\vec{a_t})_{BC} + (\vec{a_n})_{BC}$

$a_t = \alpha_2 \, BC = yc' : \alpha_2 = \frac{yc'}{BC} = \text{-----} \ rad/sec^2$ (C_b)

$a_n = \omega_2^2 \cdot BC$

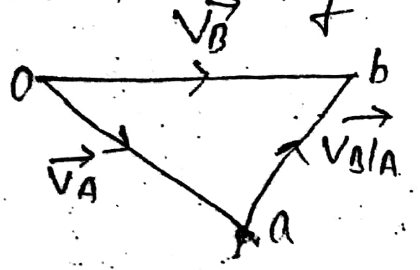
Velocity and accleration diagram of Slider crank mechanism



Then, velocity at point A w.r.t 'O'

$V_A = OA \cdot \omega$

To draw velocity diagram



$oa \perp OA$

$ob \parallel OB$

$ab \perp AB$

$ob = |\vec{V_B}| = \ldots\ldots \; m/s$

$ab = |\vec{V}_{B/A}| = \ldots\ldots \; m/s$

For angular velocity of connecting rod AB

$$|\vec{V}_{B/A}| = ab = \omega_{ab} \cdot AB$$

$$\omega_{ab} = \frac{ab}{AB} \; (\uparrow)$$

<u>For accleration</u> :

For link OA

$\alpha_{OA} = 0$ i.e $a_t = 0$

$a_n = \omega \cdot OA^2$

$$\vec{a}_n = \cancel{(\vec{a_t})_{OA}}^{\,0} + (\vec{a_n})_{on} = (\vec{a}_n)_{OA}$$

For link AB

$a_t = \alpha_{AB} \cdot AB$

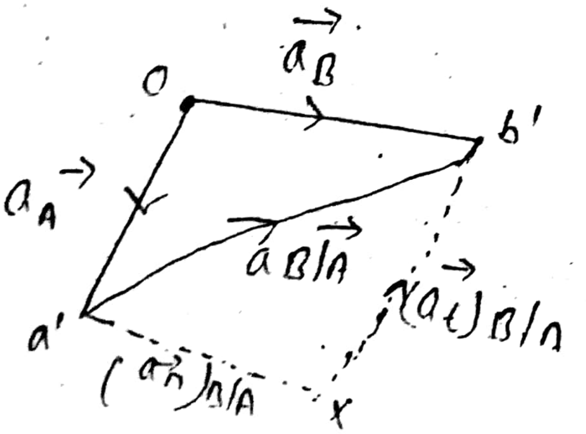
$= \omega_{AB}^2 \cdot AB$

$oa' \parallel OA$

$ob' \parallel OB$

$a'x \parallel AB$

$xb' \perp AB$



From acclesation vector diagram

$ob' = |a_B|$

$a'x = (a_n)_{B/A}$

$xb' = (a_t)_{B/A} = \alpha_{AB} \cdot AB$

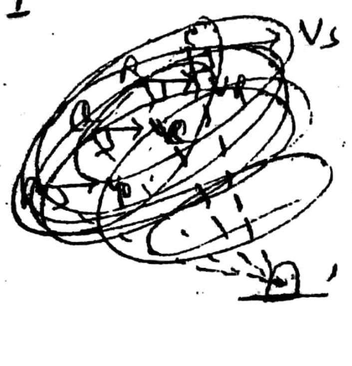
α_{AB} = Angular acclesation of connecting rod AB

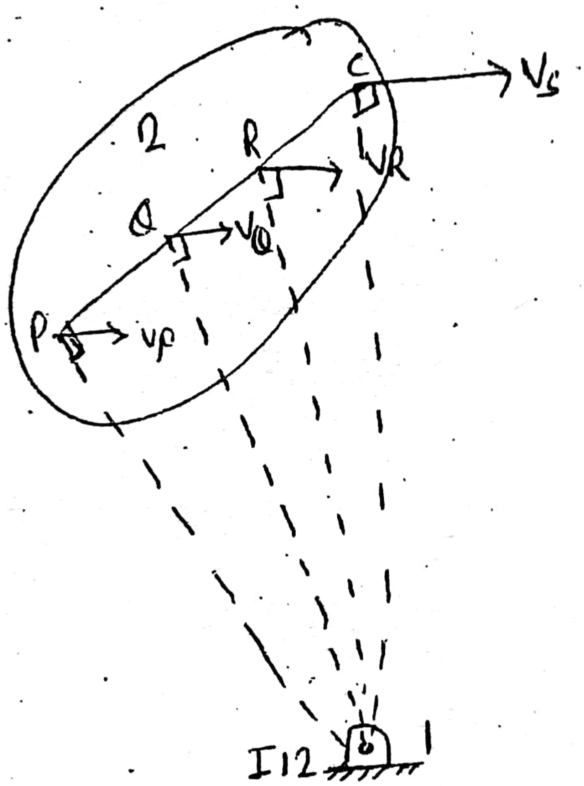
$= \frac{xb'}{AB} = \text{-----}\ rad/s^2 (\circlearrowright)$

Instanteneous centre method (IC) method

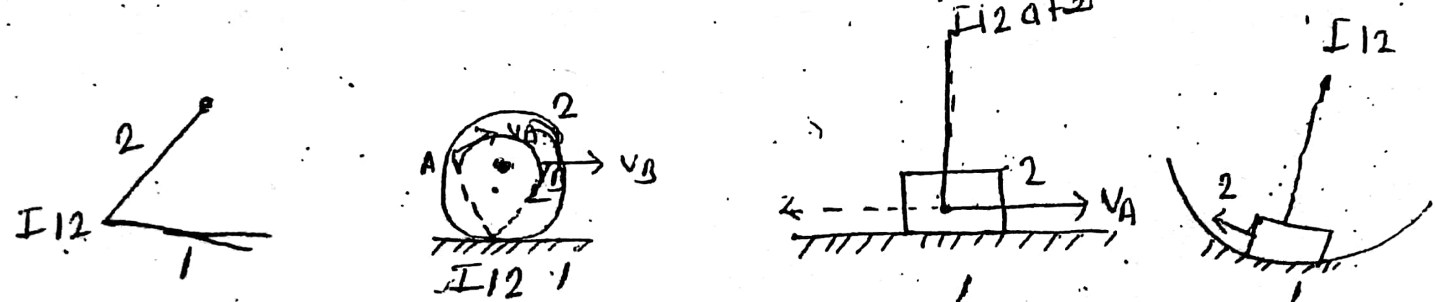
In this method, the plane motion of a rigid body is assumed as a pure rotation about a centre. Since the centre of rotation of the rigid body changes for each instant, thus this centre is called instantaneous centre of rotation.

For the relative motion of a link 2 w.r.t link 1





$$\frac{V_P}{P I_{12}} = \frac{V_Q}{Q I_{12}} = \frac{V_R}{R I_{12}} = \frac{V_S}{S I_{12}} = \omega$$



No. of instantentonous centre in mechanism

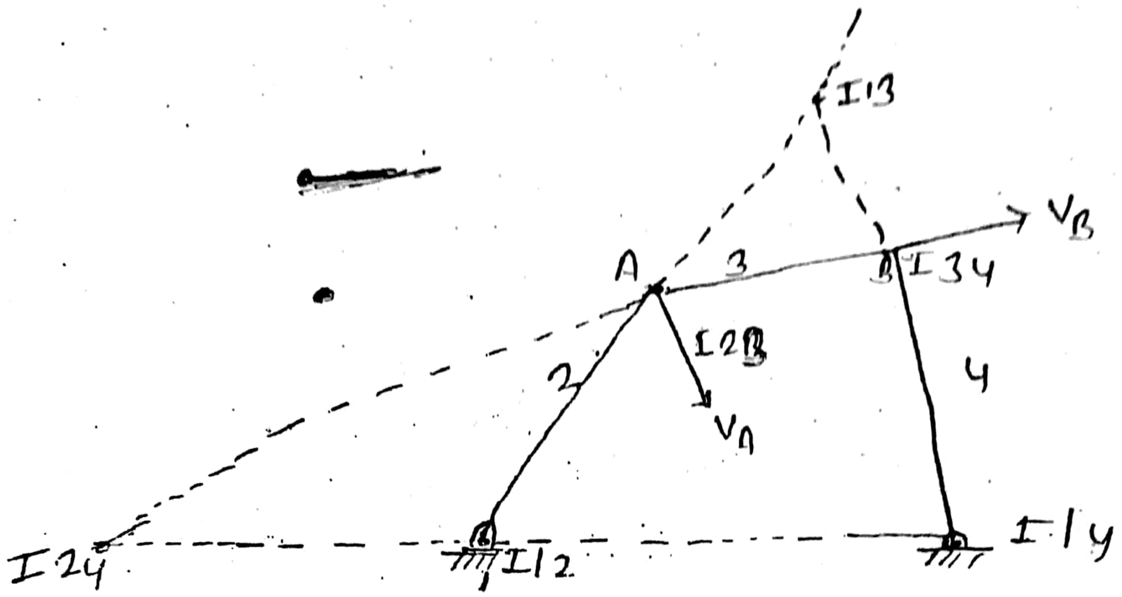
$$\text{No of I.C} = \frac{n(n-1)}{2}$$

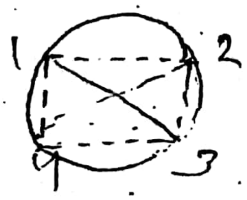
where n = no of links in mechanism

→ for four bar mechanism

$n = 4$

$$\text{No of I.C} = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$





From figure

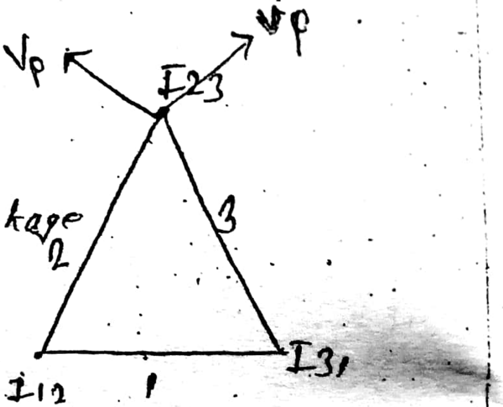
$$\frac{V_A}{I_{23}I_{13}} = \frac{V_B}{I_{34}I_{13}}$$

Fixed I.C = I12, I14
Permanent I.C → I23, I34

Neighter fixed nor permant IC = I13, I24

→ Kennedy's theorem for instantaneous centre (Three centre in line)

It states that "If the three links have relative motion to each other, then their instantaneous centre lies on a straight line."



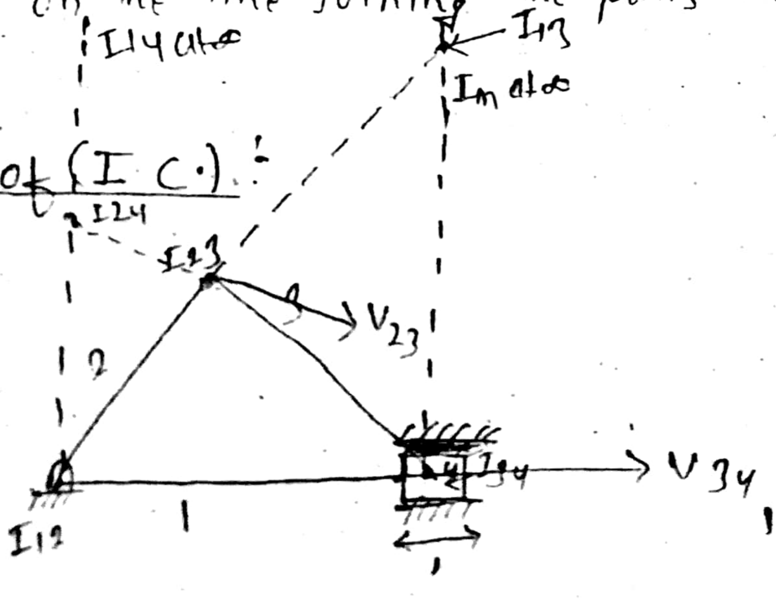
⇒ No of instantaneous centre for the linkage as shown in figure.

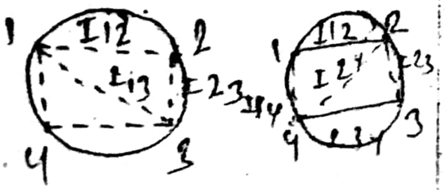
$$= \frac{n(n-1)}{2}$$

$$= 3$$

Since ~~thre~~ there is no relative velocity at the instantaneous centre, Thus to have Point I23 should lies on the line joining the points I12 and I31.

Location of (I.C.):





No of elements (n) = 4

No of I.C $= n(n-1)/2 = 6$

$$\boxed{\frac{V_{23}}{I_{23}I_{13}} = \frac{V_{34}}{I_{34}I_{13}} = \omega_3}$$

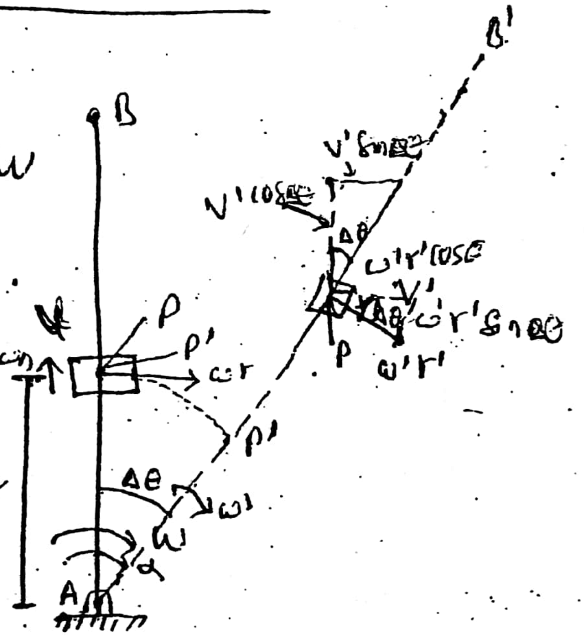
Angular velocity of crank 2 = W (?)

velocity at point I_{23} (U_{22}) = $\omega(I_{12} - I_{23})$

→ V.V.I

Coriolis component of accloration :-

Consider a link AB which rotates about point A link angular velocity w and angular accleration α and the slider slides along the link with velocity v and accloration as shown in figure.



P be a slider and p' be the link AB

$v' = v + f\Delta t$

$\omega' = \omega + \alpha\Delta t$

$r' = r + \Delta r$

For the given system

$v_p = v_{p'} + v_{p/p'}$

and

$a_p = a_{p'} + a_{p/p'}$

- Now, accloration of the slider (point p)

a_p = accleration of point p parallel to AB + accloration of point p ⊥r to AB

<u>Accleration of point p parallel to AB.</u>

Change in velocity $(\Delta v) = (v'\cos\Delta\theta - \omega' r' \sin\Delta\theta) - v$

$= (v + f\Delta t)\cdot 1 - (\omega + \alpha\Delta t)(r + \Delta r)\Delta\theta - v$

$= v + f\Delta t - \omega r\Delta\theta - \omega\cdot\Delta r\cdot\Delta\theta - \alpha r \Delta t\cdot\Delta\theta - \alpha\Delta r\,\Delta t\cdot\Delta\theta - v$

$= f\cdot\Delta t - \omega r\Delta\theta$

accloration at point p parrol to AB $= \lim_{\Delta t \to 0} \left(\frac{f\Delta t - \omega r\Delta\theta}{\Delta t}\right)$

$= f - \omega r\cdot\omega$

$= f - \omega^2 r$

Accloration of p ⊥r to AB:

Change in velocity $(\Delta v) = (v'\sin\Delta\theta + \omega' r'\cos\Delta\theta) - \omega r$

$= (v + f\Delta t)\Delta\theta + (\omega + \alpha\Delta t)(r + \Delta r)\cdot 1 - \omega r$

$= v\Delta\theta + f\Delta t\Delta\theta + \omega r + \omega\Delta r + \alpha r\Delta t + \alpha\Delta t\cdot\Delta r - \omega r$

$= V \cdot \Delta\theta + \omega \cdot \Delta r + \alpha r \cdot t$

acclerațion of point p ⊥r to A·B

$$= v\left(\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}\right) + \omega\left(\lim_{\Delta t \to 0} \frac{\Delta r}{\Delta r}\right) + \alpha r$$

$$= \alpha r + 2v\omega$$

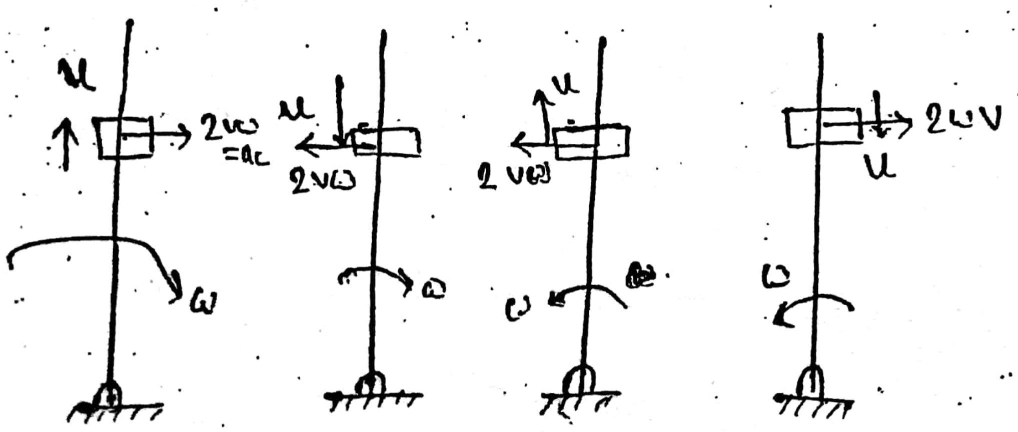
Again,

$$a_p = a_{p'} + a_{p/p'}$$

$$(f - \omega^2 r) + (\alpha r + 2v\omega) = \alpha r - \omega^2 r + a_{p/p'}$$

$$a_{p/p'} = f + 2v\omega$$

where, $2v\omega$ is called coriolis component of accloration



unit-3

Machine Design

→ Design :- decision making process
It is an iterative decision making process to create a plan by which the available resources are converted into a system that perform the required fuction and fulfill the need of human.

→ Machine design :- It is a process which proceed through several steps , evaluate the result and return to the previous steps to modify the design to make it sound design.

→ Emperical design :- - It based on past experience and existing practice
- the dimension of the main component is taken on the past experience and then the dimension of other components are taken on certain proportion of an other of main component.

- It doesnot have accuracy
- It doesnot need scientific knowledge

→ Rational design :- It is based on mathematical modeling.
→ Adaptive design,
→ Developed " :-
→ New design.

→ **Design process :**

(i) List the statement of problems of purpose for which the machine to be designed

(2) select the no. of mechanisms that satisfy the required motion. (kinematics)

(3) select suitable materials for the component (selection of material)

(4) Determine the force applied on the variours components. (Force analysis)

(5) Determine the stress (Design stress) induced in the component considering various factors (strength design)

(6) Determine the dimension of the component to prevent un due distortion (Rigidity design)

(7) modify the design on the basis of past experiences

(8) make details and assembled drawing with complete specification.

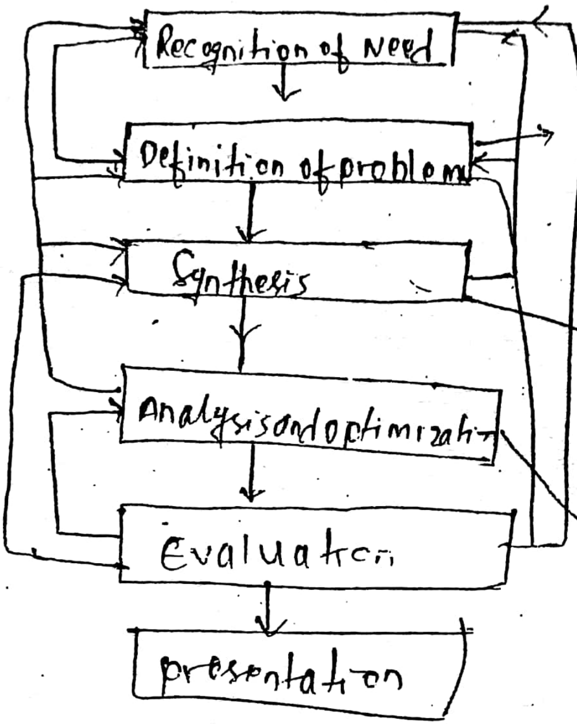
Principle stress $\sigma_1 = \dfrac{\sigma_x + \sigma_y}{2} + \dfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$

$\sigma_2 = \dfrac{\sigma_x + \sigma_y}{2} - \dfrac{1}{2}$ 〃

maximum shear stress $(\tau_{max}) = \dfrac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$

for

→ Design steps :-



specification of the component
~~specif~~
I/p, o/p · quantity, characteristics, variation of ~~variable~~ variable

select no. of proposed schemes
investigate the "
Select any one

mathematical modelling
Prototype "

⇒ Types of loading :-

- static loading
- Impulse
- cyclic loading } variable load

→ less than half vibration of natural period of vibration – impulse

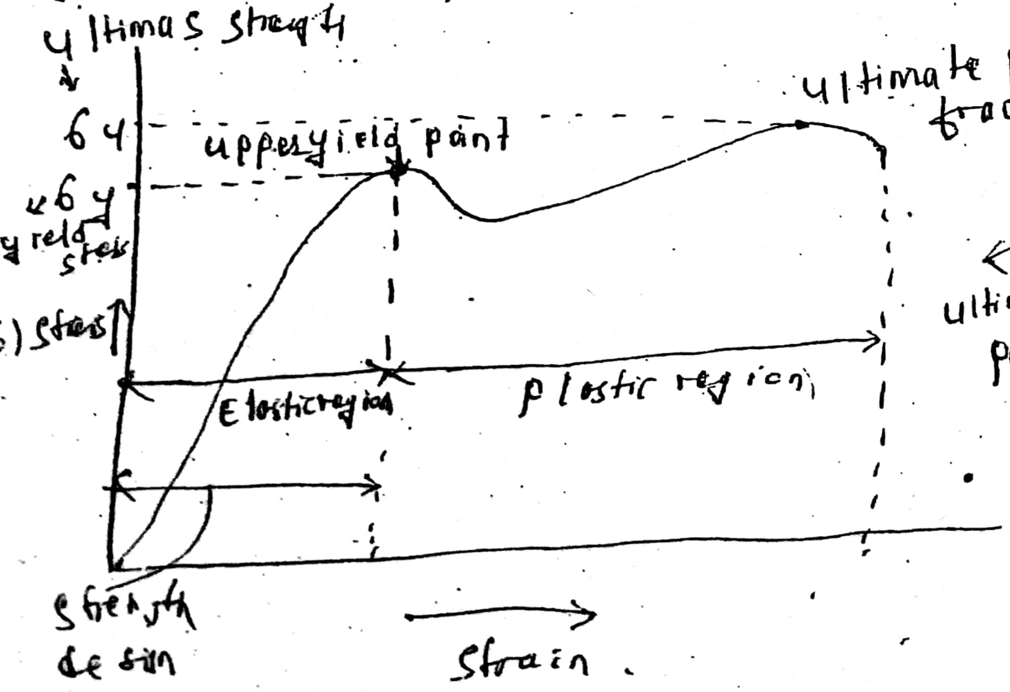
material
– excessive brittle, ductile
failure by fracture

When a member is subjected to static load, then it may fail in two ways:-

(i) Failure due to yielding (ductile materials)
(ii) Failure due to fracture (brittle materials)

The ductile material usually fail by yielding i.e permanent deformation occurs in materials. When the plastic deformation exceeds a limit, engineering purpose of the material is destroyed even through there is no fracture. For the ductile material, the limiting strength is the stress at yield point in tension or compression.

For brittle materials, the limiting strength is stress at ultimate point in tension or compression.



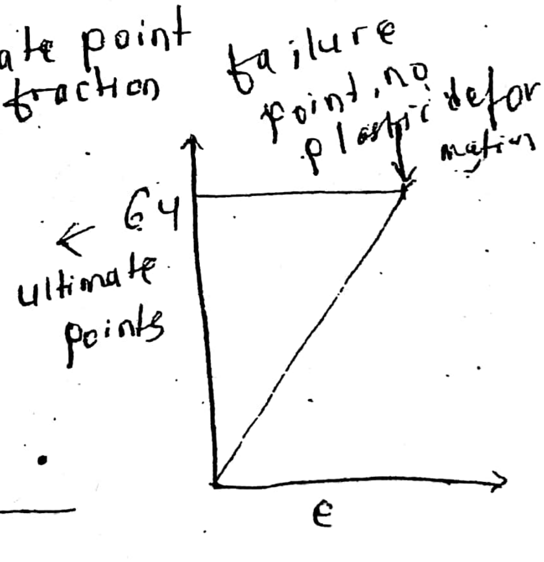


Fig: for brittle material

VVI

⇒ Theories of failure : The formal stratergies that predicts the failure of components is called theory of failure. The particular activities leading to failure is called criterion of failure i.e failure by fracture or deformation.

Following are the theories of failure

(1) Maximum principal stress theory of failure (Rankine's theory)
(2) Maximum shear stress " " " (Tresca theory)
(3) Maxm distortion energy theory " " " (Von-Mises th
(4) max m ~~principal~~ normal strain " " " "
(5) max m strain energy " " " "

(1) Max M Principal stress theory of failure (Rankine's theory

According to this theory, the failure of component occurs at a point when the maxm normal stress induced in the biaxial or triaxial of a component reaches the limitting strength of a component as determine by simple tensile test. This theory is suitable for bittle material

$$\sigma_{t1} = \frac{\sigma_{yt}}{F.S}$$, for ductile material

$$\sigma_{t1} = \underbrace{\frac{\sigma_u}{F.S}}$$, for brittle material
→ tensile ultimate stress.

where,

σ_{t1} = maxm. principal stress

σ_{yt} = yield point stress in tension
σ_u = ultimate stress

Brittle material is weak in tensile but strong in shear. This theory is very suitable for brittle materials.

(2) Maxm shear stress theory of failure:-

According to this theory, the failure occurs at the point in a member subjected to biaxial forces system where maxm shear stress induced in the member reaches the limiting shear stress of the member as determined by simple tensile strength.

$$\cancel{\tau_{max} \le \sigma_{yt}}$$

$$\tau_{max} = \sigma_{yt} / F.S \quad \text{——— (i)}$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

where, τ_{max} = max. shear stress in a bi-axial stress system

σ_{yt} = shear stress at yield point

F.S = Factor of safety

since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, then

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S}$$

This theory is mostly used for designing members of ductile materials.

(3) Maxm distortion energy theory of failure :-

According to this theory, the failure occurs at a point in a members when the maximum distorsion strain energy induced in the members subjected to biaxial force system reaches the limiting distorsion strain energy per unit volume has determined in simple test.

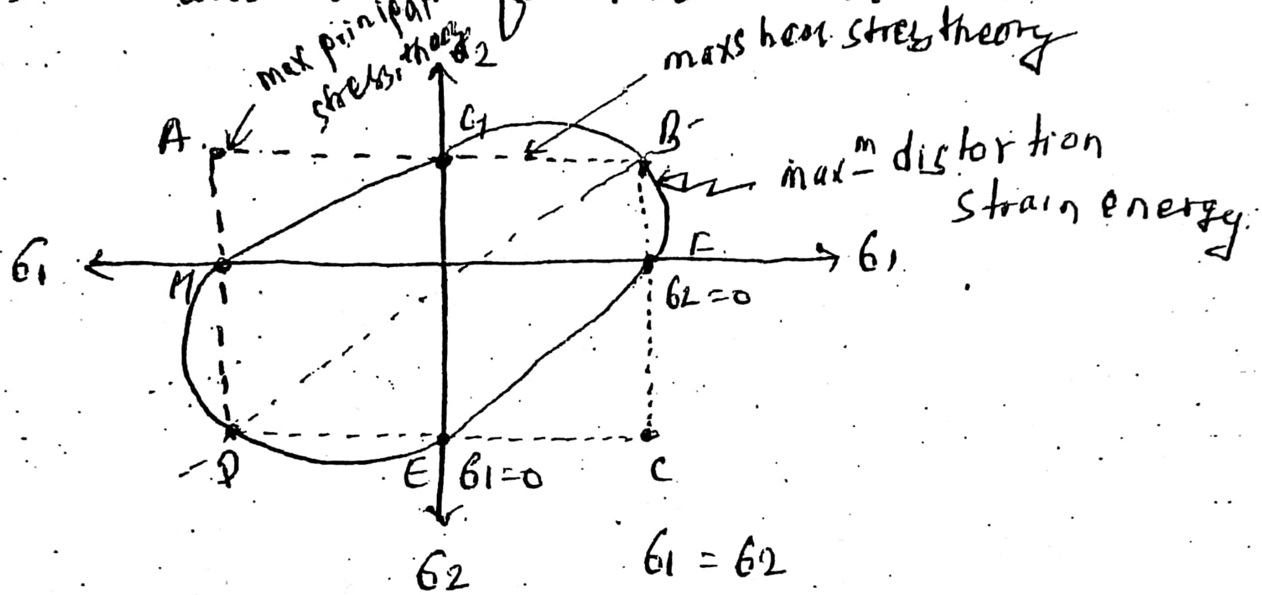
- For triaxial force system, the maxm distortion strain energy can be written as

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \leq 2\left(\sigma_y / F.S\right)^2$$

For biaxial force system : $\sigma_3 = 0$

$$\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2 \leq (\sigma_y / F.S)^2$$

This is also used for ductile material.



$$(\sigma_1 - \sigma_2) \leq \sigma_y / F.S \quad , \quad (\sigma_2 - \sigma_1) \leq \sigma_y / F.S$$

ABCD = max^m principle stress theory
EFBGHDE = max^m shear stress "

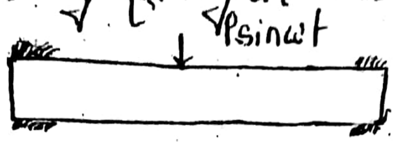
V.V.I

Fatigue :

When a member is subjected to variable load then despite the magnitude of the fluctuating stress is less than the limiting strength of the material, the failure of the materials occur after finite revolutions of the member. This phenomenon

of the reduced strength of material under variable load is called fatigue

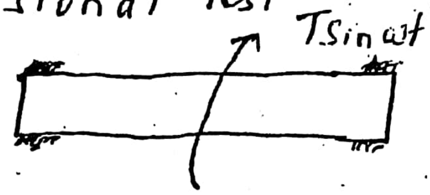
(1) Reverse bending fatigue test (endurance test)



$$\frac{M}{I} = \frac{\sigma_n}{y}$$

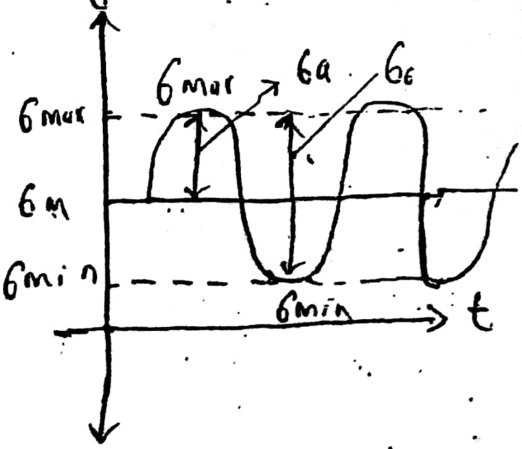
2) Reversed axial fatigue test

3) Reversed torsional test



$$F.S = \frac{\sigma_E}{\sigma_a}$$

(1) <u>Stress fluctuation</u>:



σ_{max} = maximum fluctuating stress
σ_{min} = minimum fluctuating stress
σ_m = mean stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

σ_r = stress range : $\sigma_{max} - \sigma_{min}$; σ_a = stress amplitude $= \frac{\sigma_r}{2}$

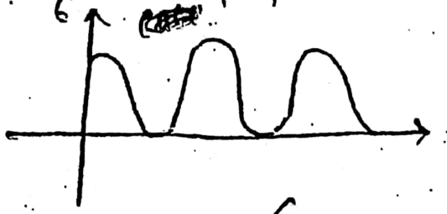
For fatigue experiment :

Case I : (fluctuating stress)

$\sigma_{max} > \sigma_{min} > 0$

$\sigma_m > 0$ (load is tensile)

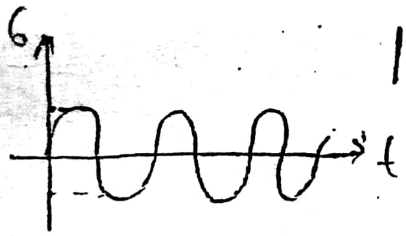
Case II : (repeated stress)



$\sigma_{max} > 0$, $\sigma_m > 0$ { repeated load (i.e tensile in nature)

$\sigma_{min} = 0$

Case III : (Completely reversed)



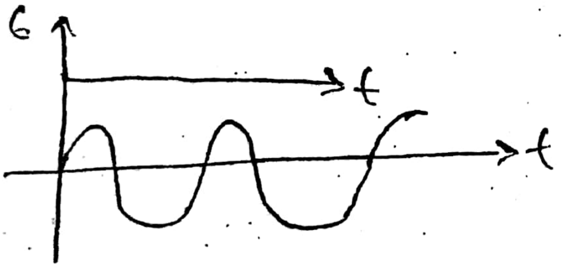
$|\sigma_{max}| = |\sigma_{min}| = |\sigma_a|$ { dangerous case for fatigue

$\sigma_m = 0$.

tensile $\rightleftharpoons$ compressive.

r = stress reversal ratio = $\frac{\sigma_{max}}{\sigma_{min}} = -1$

Case IV



$|\sigma_{max}| < |\sigma_{min}| < 0$

$\sigma_m < 0$.

(Compressive nature)

Observation :

(i) For completely revised case, most prove to easy failure

(ii) effect of σ_m

when, $\sigma_m = 0$, in case III it is most dangerious for fatigue

$\sigma_m > 0 \rightarrow$ there will be failure

$\left.\begin{array}{l}\sigma_m < 0 \\ \sigma_{max} < 0,\ \sigma_{min} < 0\end{array}\right\}$ No effect of fatigue

(iii) effect of σ_a

At the same frequency, as the stress amplitude increase it become more dangerous

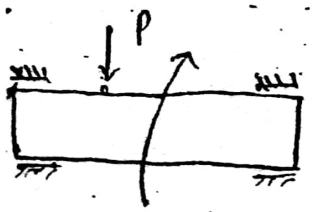
(iv) effect of frequency :

(i) hight frequency fatigue ($>10^3$) $\rightarrow$ stress influence

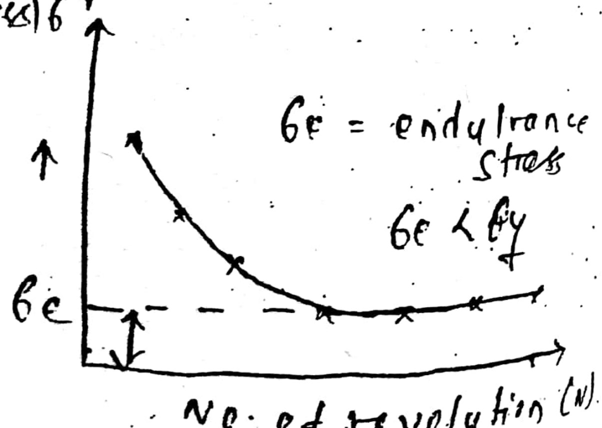
(ii) Low " " (<1000) $\rightarrow$ strain "

V.VI

Note :- Low frequency fatigue is more dangerous than high frequency fatigue, The deformation doesn't get recover completely some strain remain in the part.



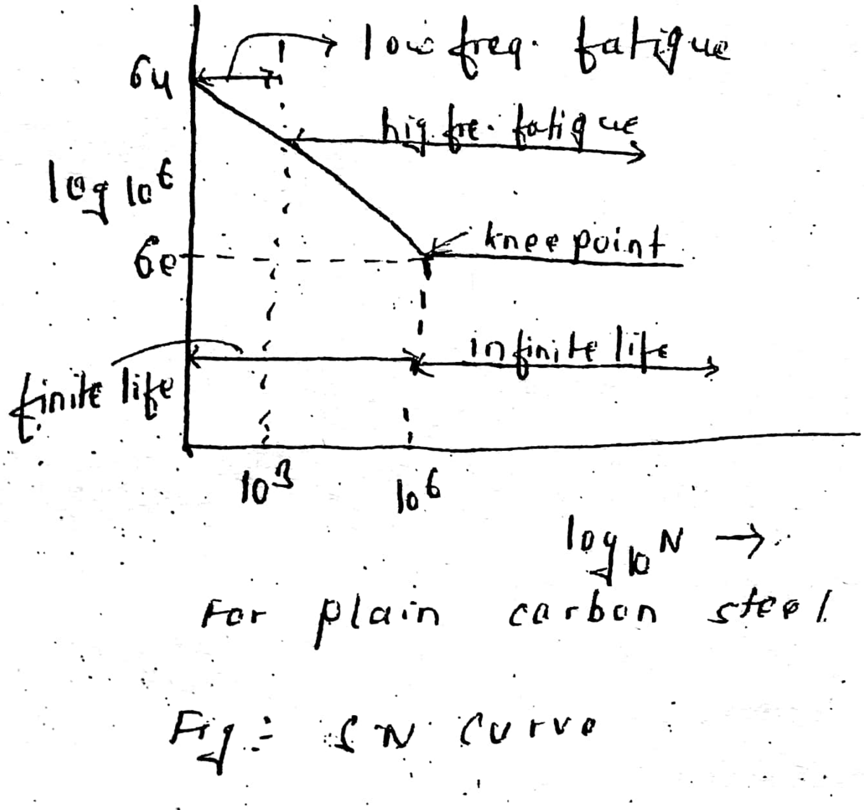
For completely reversed cyclic load



S-N curve
$\downarrow$ stress

⊛ Endurance limit / Endurance strength fatigue limit

The maxm value of reversed bending stress that can be repeated for infinite no of cycle without causing failure by progressive fracture is called endulance limit or endulrance strength. It is represented by σ_e.



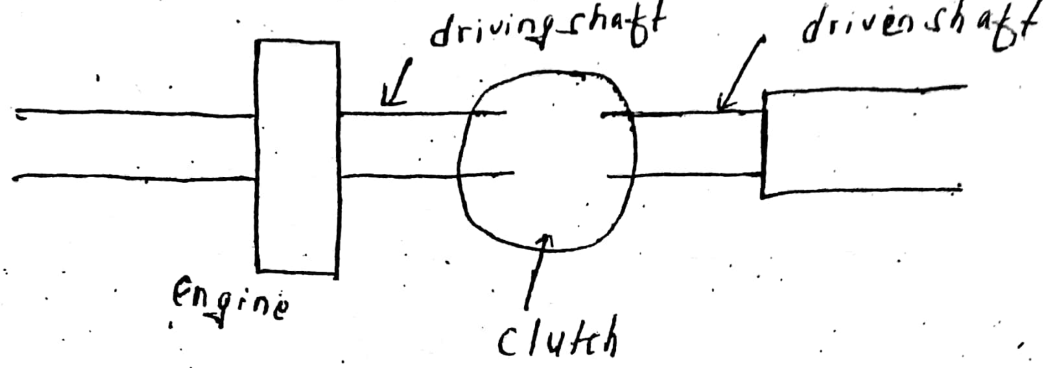
For plain carbon steel.

Fig : S-N Curve

⊛ The term endurance limit is used for reversed bending only while for other types of loading; the term endurance strength may be used when the fatigue strength of the material.

(Continue) ⟶ At last (Read more)

(8) Clutch and Brake



A clutch is the machine member used to connect the driving shaft to a driven shaft so that the driven shaft may be started or stopped at will without stopping the driving shaft. It also transmits the desired torque. The use of of a clutch is mostly found in automobiles.

→ Types of clutches:
There are two main types of clutches commonly used in engineering are:

(1) Positive clutches:
The positive clutches are used when a positive drive is required. The simplest types of a positive clutch is a jaw or claw clutch. The jaw clutch permits one shaft to drive another through a direct contact of interlocking jaws.

(2) Friction cluthes: A friction clutch has its principal application in the transmission of power of shafts and machine which must be started and stopped frequently.

Its application is also found in case which power is to be delivered to machines partially or fully loaded.

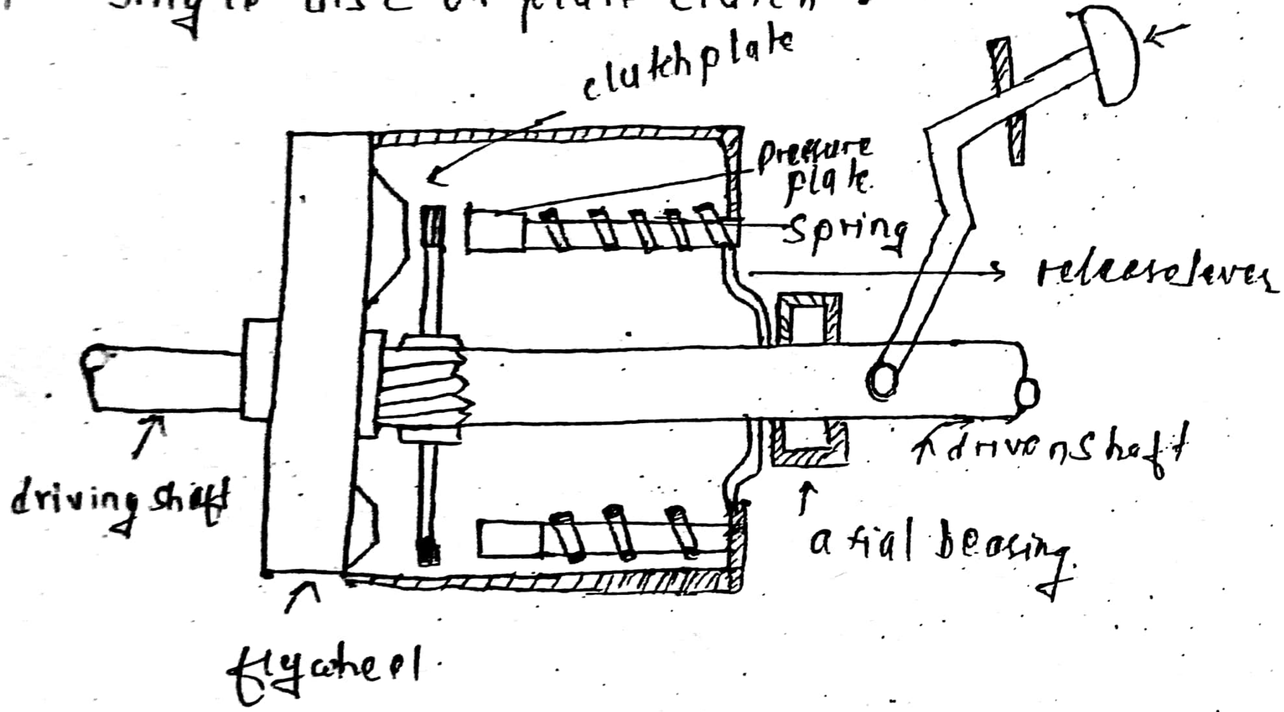
→ Types of friction cluthes

(1) Disc or plate clutches (single disc or multiple disc clutch)

(2) Cone cluthes

(3) centrifugal cluthes.

(a) Single disc or plate clutch :-

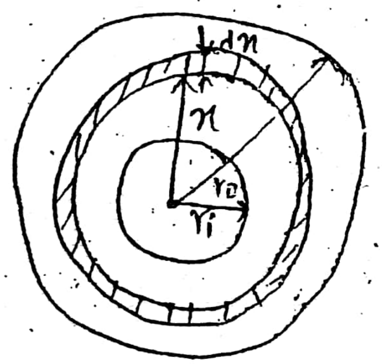


A single disc or plate cluth, as shown in fig. consists of a ~~cluth~~ clutch plate whose both sides are faced with a frictional material. It is mounted on the hub which is free to move axially along the splines of the driven shaft.

The pressure plate is mounted inside the clutch body which is bolted to the flywheel. see more page no - 888

V.V.I

Design of a Disc or plate clutch



consider a clutch plate with inner radius and outer radius of the friction linning surface. consider an elementary ring of radius n

Then,

elementary area of the friction Linning surface

$$dA = 2\pi \cdot n \, dn$$

Axial load on the elementary area $dw = P \cdot 2\pi n \, dn$

P = Pressure intensity of friction surface

r_i = inner radius of friction Lining surface

r_o = outer radius of friction " "

r = mean radius

Then,

Tangential frictional force on the clutch plate.

$$F(x) = \mu \times dw$$
$$= 2\pi \cdot \mu \cdot p \cdot x \cdot dx$$

Torque on the elementary area.

$$T_x = F(x) \cdot x$$
$$= 2\pi \mu p x^2 dx$$

In designing cluthes, two cases are considered.

(1) uniform pressure : when the pressure is uniformly distributed over the entire area of the friction face. Then the intensity of pressure

$$p = \frac{W}{\pi(r_o^2 - r_i^2)}$$

$$\therefore T_x = 2\pi \mu p x^2 dx$$

total torque transmitted by clutch plate

$$T = \int_{r_i}^{r_o} 2\pi \mu p \cdot x^2 dx$$

$$\therefore T = \left[2\pi \cdot \mu \cdot p \frac{x^3}{3} \right]_{r_i}^{r_o}$$

$$= \frac{2\pi \mu W}{\pi(r_o^2 - r_i^2)} \cdot \frac{1}{3} (r_o^3 - r_i^3).$$

$$= \mu W \cdot \frac{2}{3}\left(\frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}\right)$$

$$= \mu W \cdot r$$

where,

$$r = \frac{2}{3}\frac{(r_o^3 - r_i^3)}{(r_o^2 - r_i^2)}$$

$$\therefore T = n \mu W r$$

where n = no. of pair contact.

(2) Uniform wear:

The basic principle in designing machine parts that are subjected to wear due to sliding friction is that normal wear is proportional to the work of friction. The work of friction is proportional to the product of normal pressure P and the sliding velocity.

$\therefore$ Normal wear rate $\propto$ work of frictional force

$$P \cdot r = \text{constant } (c)$$

For elementary ring.

$$P \cdot x = c \quad \text{——— (i)}$$

$$\therefore dW = P \cdot 2\pi x \, dx$$

$$= 2\pi (P x) dx$$

$$= 2\pi c \, dx \quad (\text{from } 0$$

$$\therefore W = \int_{r_i}^{r_o} 2\pi c \, dx$$

$$= 2\pi c(r_o - r_i)$$

$$\therefore \quad c = \frac{W}{2\pi(r_o - r_i)}$$

$$\text{total torque } (T) = \int_{r_i}^{r_o} 2\pi \mu p x^2 \, dx$$

$$= \int_{r_i}^{r_o} 2\pi \mu (px) x \, dx$$

$$= 2\pi\mu \cdot c \int_{r_i}^{r_o} x \, dx$$

$$= 2\pi\mu \cdot \frac{W}{2\pi(r_o - r_i)} \cdot \frac{r_o^2 - r_i^2}{2}$$

$$= \mu W \cdot \frac{r_o + r_i}{2}$$

$$= \mu W r_o$$

$$\text{where, } r = \frac{r_o + r_i}{2}$$

for, n = pair of surface contact

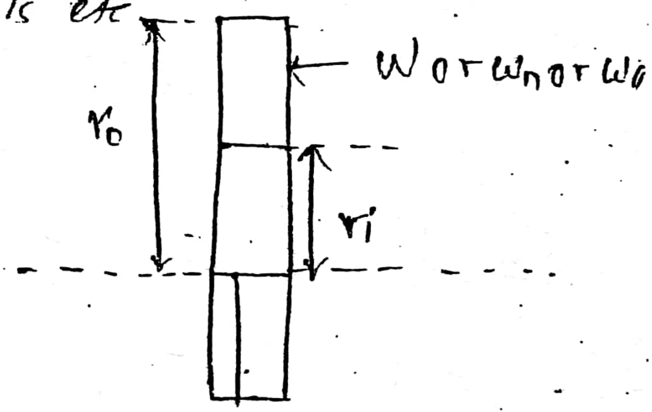
$$\boxed{T = n \mu W r}$$

Note :- (a) In case of a new clutch, the intensity of pressure is approximately uniform, but in an old clutch the uniform wear theory is more approximate.

(b) The uniform pressure theory give a higher friction torque than the uniform wear theory. Therefore in case of friction clutches. Uniform wear should be considered; unless otherwise stated.

(b) multiple Disc clutch :

A multiple disc clutch may be used when a large torque is to be transmitted. The inside discs are fastened to the driven shaft to permit axial motion. The multiple disc clutches are extensively used in motor cars, machine tools etc.



Let, n_1 = Numbers of discs on the driving shaft,
n_2 = Numbers of discs on the driven "

∴ numbers of pair contact surface $n = n_1 + n_2 - 1$

Total ~~fro~~ frictional ~~fro~~ torque

$$T = n \mu W \cdot r$$

where r = mean radius.

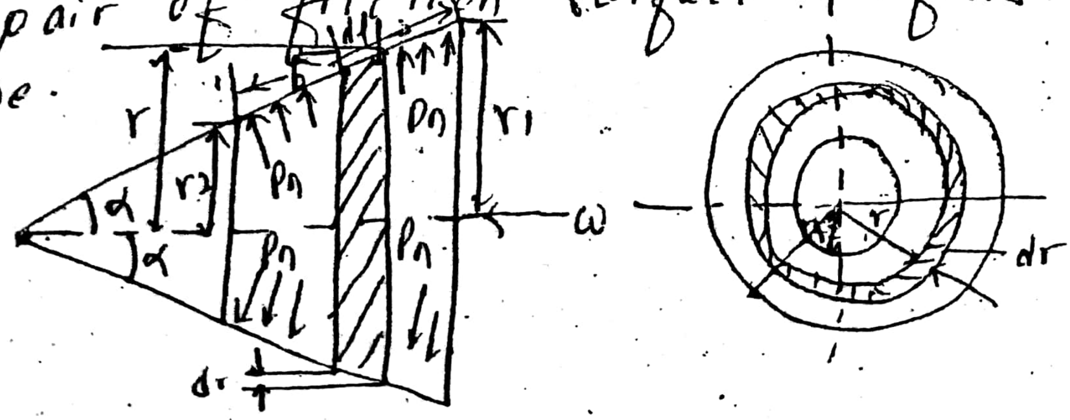
$$r = \frac{2}{3} \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2} \rightarrow \text{For uniform pressure}$$

$$= \frac{r_o + r_i}{2} \quad \text{for uniform wear.}$$

(2) Cone clutch :- A cone clutch was extensively used in automobiles, but now-a-day it has been replaced by disc clutch. Read theory from page no: 902 - 903 also fig (24.6).

→ Design of a cone clutch :-

considers a pair of friction surface of a cone clutch. A little considera tion will show that the area of contact of a pair of friction surface is a frustrum of cone.



consider a small ring of radius r and thickness dr as shown in fig. Let dl is the length of ring of the friction surface

$$dl = dr \cos\alpha$$

$$\text{Area of ring} = 2\pi r\, dl$$
$$= 2\pi r\, dr \operatorname{cosec}\alpha$$

case (1)

(a) uniform pressure,

normal force on ring $(\delta W_n) = P_n \times 2\pi r\, dr \operatorname{cosec}\alpha$

$$\text{axial force } (\delta W) = \delta W_n \times \sin\alpha$$
$$= P_n \times 2\pi r \cdot dr \operatorname{cosec}\alpha \times \sin\alpha$$
$$= 2\pi \times P_n \cdot r \cdot dr$$

∴ Total load transmitted to the clutch or the axial spring force required.

$$W = \int_{r_2}^{r_1} 2\pi \times P_n \cdot r \cdot dr = 2\pi P_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

Direct start from

$$= \pi p_n \left[r_1^2 - r_2^2\right]$$

$$\boxed{P_n = \frac{W}{\pi\left((r_1)^2 - (r_2)^2\right)}} \qquad \text{—— (i)}$$

we know that frictional force on the ring acting tangentially at radius r,

$$F_r = \mu \cdot \delta w_n$$

$$= 2\pi r\, dr \operatorname{cosec}\alpha$$

$$\therefore T_r = F_r \times r$$

$$= 2\pi \mu \cdot p_n \operatorname{cosec}\alpha\, r^2\, dr$$

Now,

$\therefore$ total frictional torque

$$T = \int_{r_2}^{r_1} 2\pi \mu\, p_n \cdot \operatorname{cosec}\alpha\, r^2\, dr$$

$$= 2\pi \mu \cdot p_n \cdot \operatorname{cosec}\alpha \cdot \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi \mu\, p_n \operatorname{cosec}\alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of P_n from equation (i) we get

$$T = 2\pi\mu \frac{W}{\pi((r_1)^2-(r_2)^2)} \times \operatorname{cosec}\alpha \left(\frac{(r_1)^3-(r_2)^3}{3}\right)$$

$$\boxed{T = \frac{2}{3}\mu \cdot W \operatorname{cosec}\alpha \left[\frac{(r_1)^3-(r_2)^3}{(r_1)^2-(r_2)^2}\right]}$$

(b) Considering Uniform wear :-

$$P_r \cdot r = \text{Costant} \quad (C)$$

$$P_r = C/r$$

~~Now,~~

$$\delta W_n = P_r \times 2\pi r\, dr \cdot \operatorname{cosec}\alpha$$

axial force acting on the ring.

$$\delta W = \delta W_n \times \sin\alpha$$

$$= 2\pi \times p_r \cdot r\, dr$$

$$= 2\pi \times \frac{C}{r} \times r \cdot dr = 2\pi \cdot C \cdot dr$$

∴ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi C \cdot dr = 2\pi C \left[r\right]_{r_2}^{r_1}$$

$$= 2\pi C (r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)} \quad \text{——— (iii)}$$

we know that frictional force on the ring ~~actu~~ acting tangentially at radius r_1

$$F_r = \mu \cdot \delta W_n = \mu \cdot p_r \times 2\pi r\,dr \cdot \operatorname{cosec}\alpha$$

$\therefore$ frictional torque acting on the ring.

$$T_r = F_r \times r$$

$$= \mu \cdot p_r \times 2\pi r\,dr \cdot \operatorname{cosec}\alpha \times r$$

$$= \mu \times \frac{C}{r} \times 2\pi r\,dr \cdot \operatorname{cosec}\alpha \times r$$

$$= 2\pi \mu \cdot C \cdot \operatorname{cosec}\alpha \times r\,dr$$

Integrating this expression within the limit from r_2 to r_1 for the total frictional torque on the clutch.

$\therefore$ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi \mu \cdot C \operatorname{cosec}\alpha \times r\,dr$$

$$= 2\pi\mu C \cdot \operatorname{cosec}\alpha \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

T ~~$= \mu W \operatorname{cosec}$~~ Sub. the valu of

c from equation (11)

$$T = \mu W \operatorname{cosec}\alpha \left(\frac{r_1+r_2}{2}\right)$$

$$= \mu W R \operatorname{cosec}\alpha$$

where $R = \frac{r_1+r_2}{2}$ = mean radius of friction surface.

Brake:

It is a device which is used to appl. external frictional force to the moving body to stop or retard it by absorbing the kinetic and potential energy in the form of heat which must be dissipated rapidly. It is used to regulal or control the motion of moving body.

⟹ Types of brakes:

According to the means used for transforming the energy by the braking element are classified as:

(1) Hydraulic brakes: eg: pumps or hy Aerodynamic brak

(2) Electric brakes: generator and eddy current brakes

(3) Mechanical brakes:

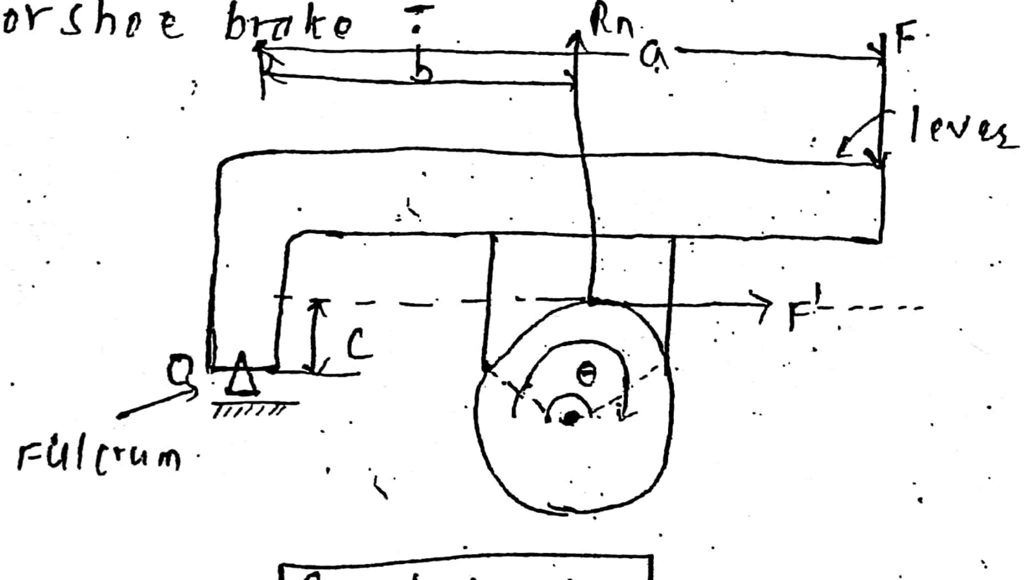
According to the direction

of acting force, may be divided into the following two group.

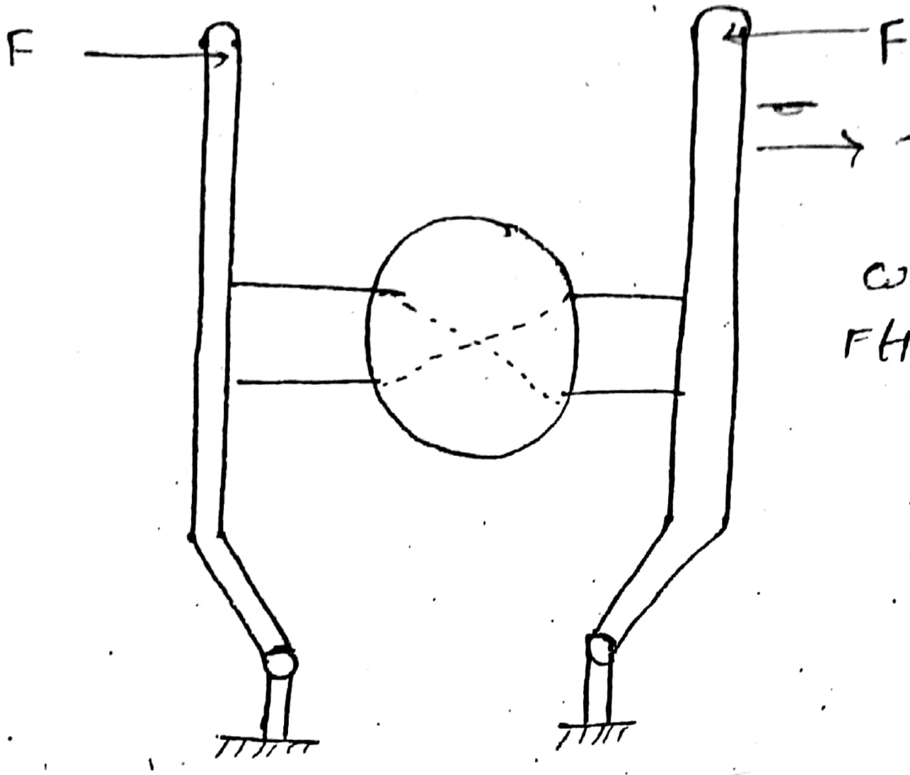
(a) Radial brakes :- In these brakes, the force acting on the brake drum is in radial direction. The radial brakes may be sub-divided into external brakes and internal brakes. According to the shape of the friction element, these brakes may be block or shoe brake and band brakes.

(b) Axial brakes :- In these brakes, the force acting on the brake drum is in axial direction. The axial brakes may be disc brakes and cone brakes.

Block or shoe brake :-



Single block.



$\rightarrow T_B = (F_{t1} + F_{t2}) r \rightarrow$ braking torque

where

F_{t1} and F_{t2} are the braking force.

Double block

For given figure, F = Fluctuating force applied at end of lever

R_n = Normal reaction applied by drum on the block

F' = Frictional force on the block at contact surface

μ = coeff. of friction

T = torque.

Then,

Torque $(T) = F' r = \mu R_n r$ where

r = radius of drum

θ : contact angle

$$\text{If } \theta > 45^\circ \; \mu' = \mu \left(\frac{4 \sin \theta/2}{\theta + \sin \theta} \right)$$

If $\theta > 45°$ Then effective radius of the drum.

$$h = r\left(\frac{4\sin\theta/2}{\theta + \sin\theta}\right)$$

$$T = \mu R_n h$$

$$= \mu R_n r\left(\frac{\sin\theta/2}{\theta + \sin\theta}\right)$$

$$= \mu' R_n r$$

Case (a) when the fulcrum of the level passed through the line of action of frictional force.

For clockwise direction or anticlockwise rotation of drum. taking momemt about point O

$$Fa = R_n b$$

$$R_n = \frac{Fa}{b}$$

$$\text{Torque } (T) = \mu R_n r$$
$$= \mu F r\, a/b$$

(b) when the fulcrum of the level is below the line of action of frictional force.

For clockwise direction,

Taking moment about 'O'

$$Fa + F'c = R_n b$$

$$Fa = R_n b - F'c$$
$$= R_n b - \mu R_n c$$
$$= R_n (b - \mu c)$$
$$\therefore F = \frac{R_n}{a}(b - \mu c)$$

For anticlockwise rotation of drums,

$$F = \frac{R_n (b + \mu c)}{a}$$

Frictional force is added to the applied force, hence it is called self ~~engery~~ energised brake.

$$F = \frac{R_n}{a}(b - \mu c)$$

If $b - \mu c = 0$

$b \leq \mu c$ when drum comes in contact with block, the brake is applied itself. It is called self locked brake.

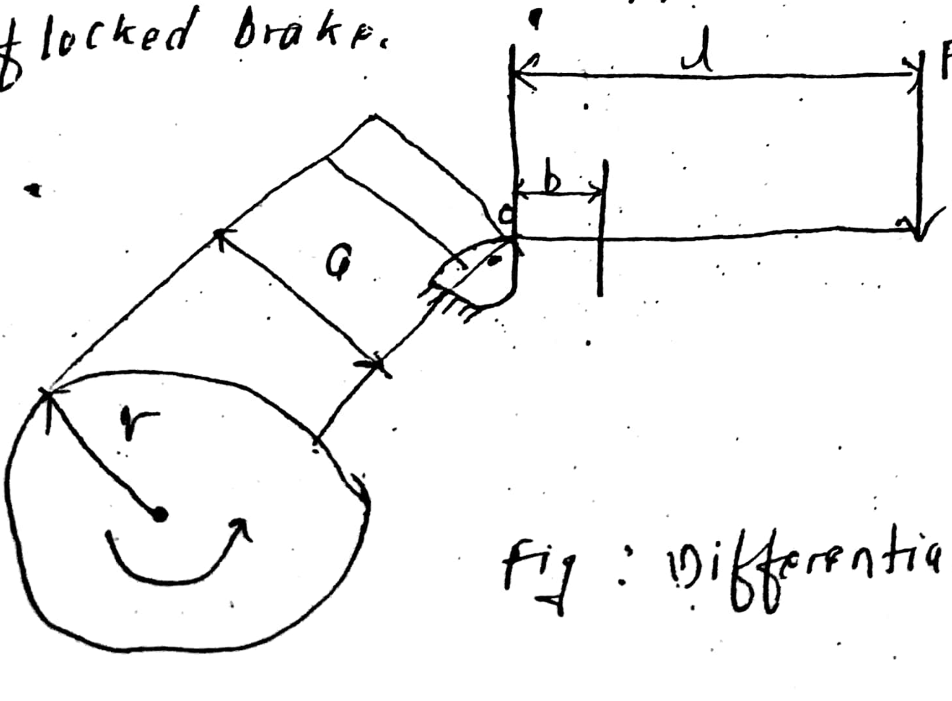
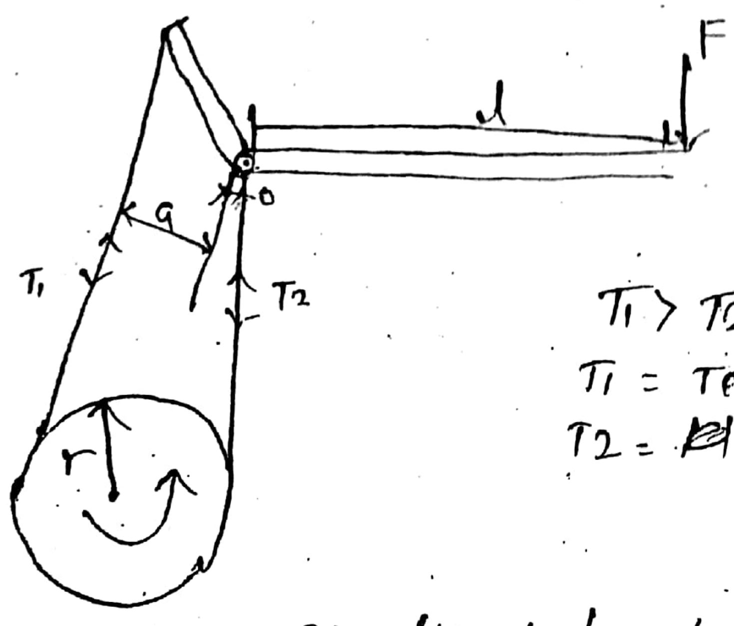


Fig : Differential band brake.



$T_1 > T_2$

T_1 = Tension on tight side

T_2 = " on slack side

Fig Simple band brake

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta} \qquad T_2 = T_1 / e^{\mu\theta}}$$

Torque developed at contact surface

$$T = (T_1 - T_2)\, r$$

For given figure, when the drum rotates in anticlockwise direction

Taking moment about point O

$$T_1 \times a = F \times d$$

$$T_1 = \frac{F \times d}{a}$$

we know $T = (T_1 - T_2)\, r$

$$T = \left(T_1 - \frac{T_1}{e^{\mu\theta}}\right) r$$

$$= T_1 \left(\frac{e^{\mu\theta} - 1}{e^{\mu\theta}}\right) r$$

$$= \frac{Fd}{a}\left(\frac{e^{\mu\theta}-1}{e^{\mu\theta}}\right)r$$

For clockwise rotation,

Taking moment about point 'O',

$$T_2 \times a = F \times d$$

$$T_2 = \frac{Fd}{a}$$

$$\therefore \quad T = (T_1 - T_2)\, r$$

$$= (T_2 e^{\mu\theta} - T_2)\, r$$

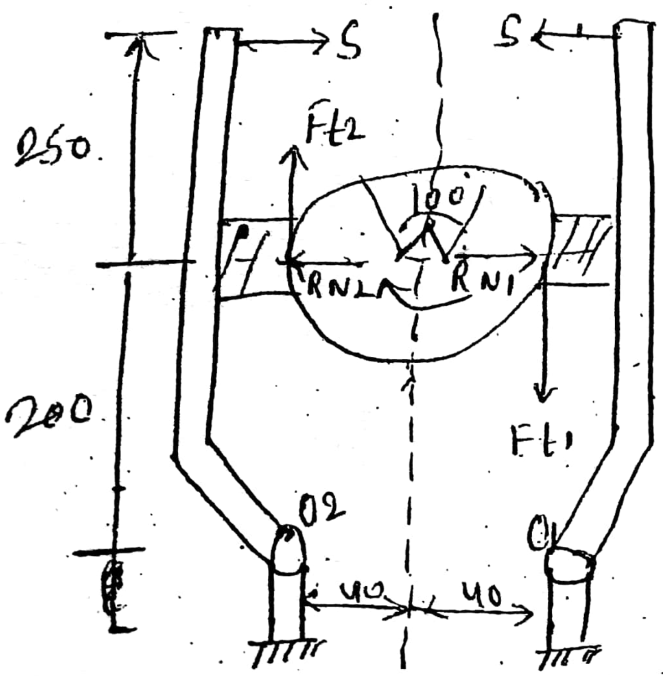
$$= T_2 (e^{\mu\theta} - 1)\, r$$

$$\therefore \boxed{T = \frac{Fd}{a}(e^{\mu\theta} - 1)\, r}$$

V.V.I.I Numerical :

① A double shoe brake shown in figure is capable of absorbing a torque of 1400N-m. The diameter of the drum is 350 mm and the angle of contact of each shoe is 100°. If the coefficient of friction between the brake drum and lining is 0.4. Find (i) the spring force necessary to set the brake (ii) The width of the brake shoes. If the bearing pressure on the lining material is not exceed of 0.3 N/mm^2

Soln



Soln.

Here given,

$T_B = 1400$ N-m $= 1400 \times 10^3$ N-mm

$d = 350$ mm $\therefore (r) = 175$ mm ;

$2\theta = 100° = 100 \times \pi/180$
$= 1.75$ rad

$\mu = 0.4$ $\quad p_b = 0.3\ N/mm^2$

(I) Spring force necessary to set the brake.

From above figure since angle of contact is greater than 60°, then,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.4 \times \sin 50°}{1.75 + \sin 100} = 0.45$$

Taking moment about O_1 we have,

$$S \times 450 = R_{N1} \times 200 + F_{t1}(175 - 40)$$

$\boxed{R_{N1} = F_{t1}/\mu'}$

$$= \frac{F_{t1}}{0.45} \times 200 + F_{t1} \times 135$$

$$= 579.4 F_{t1}$$

$$\therefore F_{t1} = S \times 450 / 579.4 = 0.7765$$

Again, taking taking moment about O_2 we have

$$S \times 450 + F_{t2}(175 - 40) = R_{N2} \times 200$$

$$S \times 450 + F_{t2} \times 135 = \frac{F_{t2}}{0.45} \times 200 = 444.4 F_{t2}$$

$$444.4 F_{t2} - 135 F_{t2} = S \times 450$$

$$F_{t2} = 1.454 S$$

Now, ~~torq~~ torque capacity of the brake(T_B)

$$1400 \times 10^3 = (F_{t1} + F_{t2}) \times r$$

$$S = \frac{1400 \times 10^3}{390.25} = 3587 N \text{ Ans}$$

(7) width of brakeshoes

unit-3 — Continue

⇒ **Stress Concentration factor :**

$$\sigma = F/A$$

The irregular distribution of stress due to surface changes is called stress concentration. It is mainly due to holes, notch, surface roughness, fillets, spline, key ways present in the member. A stress concentration factor is the ratio of maxm stress at discontinuous surface to the nominal stress at same section.

→ **stress concentration factor :-**

It is the ratio of stress at irregular surface or section (i.e notch hole etc) to the nominal stress at the section. It is denoted by k_t

$$k_t = \frac{\text{max}^m \text{ stress at discontinuous section}}{\text{nominal stress}}$$

$$k_t = \frac{\sigma_{max}}{\sigma_0}$$

The magnitude of stress concentration factor (k_t) → depends on.

(a) geometry of surface
(b) types of loading
(c) " " material

member (at a notch or fillet) to the nominal stress ~~at~~ to the nominal stress at the same section based upon net area.

⇒ For Fatigue stress concentration factor:

For cyclic loading, fatigue stress concentration factor is used instead of theortical stress concentration factor.

Fatigue stress concentration factor (k_f)

$$k_f = \frac{\text{Endurance strength of plain specimen}}{\text{Endurance strength of notched ''}}$$

k_f and k_t can be combined by notch sensitivity factor (q)

$$q = \frac{k_f - 1}{k_t - 1} \Rightarrow \boxed{k_f = 1 + q(k_t - 1)}$$

⇒ Factor to consider while desiging machine parts to avoid fatigue failure.

(a) the variation in the size of the component should be as gradual as possible

(b) holes, notches and other stress raisers should be avoided

(c) proper stress de-concentrators such as fillets and notches should be provided.

(d) Parts should be protected from corrosive atmosphere

(e) smooth finish of outer surface of the component increase the fatigue life

(f) material with high fatigue strength should be selected.

V.V.I

⇒ Fatigue Design : (See sir note)

There are three method for fatigue design:

(1) Soderberg method → yield pt. is taken as failure criterion

(2) Goodman method

3 Gerber method method } → ultimate

→ unit -(4) (numerical-based topic)
Shaft- axle, keys and shaft couplings

Shaft:-
A shaft is a rotating machine element which is used to transmit power from one place to another. Since it may be subjected to bending, axial, torsional load. The bending stress, shear stress are induced in it. It transmit bending as well as torsional moment.

Types of shaft:-
According to function.
1. Transmission shaft:- These shafts transmit power between the source and the machine absorbing power. These shaft carry machine parts such as pulleys, gears etc, therefore they are subjected to bending in addition to twisting.
2. Machine shaft:-
These shaft form an integral parts of the machine itself. The crank shaft is an examples of machine shaft.

V.V.I

Design of Shafts :

The shaft may be designed on the basis of

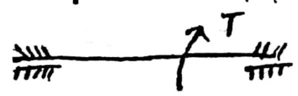
(1) Strength (2) Rigidity and stiffness

In designing shaft on the basis of strength, the following case may be considered :-

→ Shafts subjected to torsional moment or torque only
→ " " " bending " "
→ " " to combined twisting and bending moments.
→ Shafts subjected to axial load in addition to combined torsional and bending loads.
→ " Subjected to variable load.

(a) Shafts subjected to torsional moment or shear stress only.

When the shaft is subjected to a twisting moment only, then the shaft may be obtained by using the torsion equation. We know that,



$$\frac{T}{J} = \frac{\tau}{r} \quad \text{——— (i)}$$

where, T = Twisting moment = Torque

J = polar M.I $= \frac{\pi d^4}{32}$

τ = Torsional shear stress

r = Distance from neutral axis to the

Then, we know,

$$J = \pi/_{32} \times d^2 \quad \text{— (ii)}$$

Then eq (i) become

$$\boxed{T = \frac{\pi}{16} \times \tau\, d^3}$$

Note: For hollow shaft.

$$T = \pi/_{16} \times \tau (d_o)^3 (1 - k^4)$$

d_i = inside diammeter
d_o = outside "

(2) Shaft subjected to bending moments only.
When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that,

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

M = bending moment
I = M. of Inertia $= \frac{\pi d^4}{64}$ for shaft
σ_b = bending stress
y = Distance from neutral axis.

Then,

$$\boxed{M = \pi/32 \times \sigma_b \times d^3}$$

Note:

(i) For hollow shaft,

$\cancel{M = \pi/32 \times \sigma_b (d_o)^3 (1-k^4)}$

$$I = \pi/64 \, (d_o^4 - d_i^4)$$

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

or,

$$M = \frac{\pi}{64} (d_o^4 - d_i^4) \frac{\sigma}{d_o/2}$$

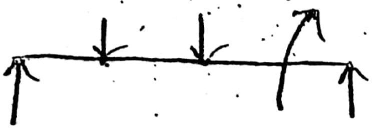
$$= \frac{\pi}{32} \sigma_b (d_o)^3 \left(1 - \frac{d_i^4}{d_o^4}\right)$$

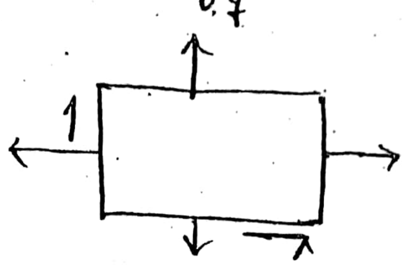
$$\boxed{M = \pi/32 \, \sigma_b \times (d_o)^3 (1-k^4)}$$

where $k = \frac{d_i}{d_o}$

It is used to get dimention for axle.

3) Shaft subiected to combined bending and twisting moment





7) On the basis of Maximum shear stress theory of failure:-

The max^m shear stress induced in the shaft due to combined bending and torsional load is given by,

$$\tau_{max} = \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

$$= \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32m}{\pi d^3}\right)^2 + 4\cdot\left(\frac{16\cdot T}{\pi d^3}\right)^2}$$

$$= \frac{1}{2}\cdot\frac{32}{\pi d^3}\sqrt{m^2 + T^2}$$

$$\therefore \tau_{max} = \frac{16}{\pi d^3}\sqrt{m^2 + T^2}$$

This τ_{max} is considered as allowable shear stress for the shaft.

$$\tau = \frac{16}{\pi d^3}\sqrt{M^2+T^2}$$

or, $$\sqrt{M^2+T^2} = \frac{\pi}{16}\tau d^3 = Te$$

Where, Te = equivalent torque

$$\therefore \boxed{Te = \frac{\pi}{16}\tau d^3}$$

(b) According to max^m principal (normal) stress theory of failure,

$$\sigma_{max} = \sigma/2 + \sqrt{\left(\frac{\sigma}{2}\right)^2+\tau^2}$$

$$= \frac{1}{2}\left(\frac{32M}{\pi d^3}\right) + \frac{1}{2}\sqrt{\left(\frac{32M}{\pi d^3}\right)^2 + 4\left(\frac{16T}{\pi d^3}\right)^2}$$

$$\sigma_{max} = \frac{32}{\pi d^3}\left[\frac{M}{2} + \frac{1}{2}\sqrt{M^2+T^2}\right]$$

Considering σ_{max} as allowable bending stress for the material of shaft,

$$\sigma = \frac{32}{\pi d^3}\left[\frac{M}{2} + \frac{1}{2}\sqrt{M^2+T^2}\right]$$

$$\boxed{\frac{1}{2}\left(M+\sqrt{M^2+T^2}\right) = \frac{\pi}{32}\sigma d^3 = Me}$$

where Me = equivalent bending moment

(2

1) Shaft subjected to variable load :-

For shaft subjected to variable load, the shock and fatigue factor can be taken,

$$Me = \frac{1}{2}\left[km \cdot M + \sqrt{km \cdot m^2 + kt \cdot T^2}\right]$$

where, km = shock and fatigue factor for bending load

kt = " " " " twisting ~~load~~.

$$\boxed{Te = \sqrt{km \cdot m^2 + kt \cdot T^2}}$$

→ Design of shaft on the ~~base~~ basis of Rigidity

(i) Torsional rigidity :- The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the values would be effected.

The torsional deflection may be obtained by using the torsion equation

$$\frac{T}{J} = \frac{G \cdot \theta}{L} \quad \text{or} \quad \theta = \frac{T \cdot L}{J \cdot G}$$

$$J = \pi/32 \times d^4 \quad \text{— For solid shaf}$$

$$= \pi/32 \left[(do)^4 - (di)^4\right] \quad \text{" hollow "}$$

2) Lateral rigidity :- It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces.

when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for elastic curve of a beam i.e

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

V.V.I

Shaft (Numerical :- page no :- 513 to 535

Example :- 14.1, 14.2, 14.3, 14.4, 14.5, 14.6, 14.7, 14.8, 14.9, 14.10, 14.11, 14.12, 14.13, 14.14 and 14.15

key :- A piece of mild steel inserted between ~~the~~ shaft and hub or boss of the pully to connect these together in order to prevent relative motion between them.

It is a power transmission element.

The main functions of key are :-

(i) to provide temporary joint between shaft and other element mounted unit.

~~(ii) to provide relative ... between elements.~~

(ii) to transmit required torsional moment.

<u>Classification</u>

Key

- Light duty
  - Square key
  - Rectangular "
  - Gibhead "
  - wood-ruff key
  - Flat key
  - Saddle key
- Heavy duty
  - Round keys
  - Barth key
  - Kennedy "
  - Tangent "
  - Feather "
  - Spline "

→ **Types of keys :-** The very important types are :-

$W/3 = d/6$

~~(1)~~ Sunk key (Mainly)
(2) saddle key
(3) Tangent key
(4) Round key
(5) Splines "

(1) Sunk keys :- The sunk keys are provided half in the key way of the shaft and half in the key ways of the hub or boss of the pully.

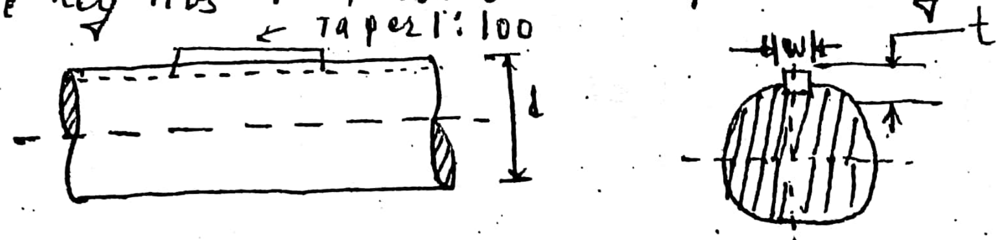
Types of sunk key

(a) Rectangular :- A rectangular sunk key is shown in fig below. The usual proportions of this key are :

width of key, $w = d/4$; and thickness of key, $t = 2w/3 = d/6$

where, d = Diameter of the shaft or diameter of the hole in the hub

The key has 1 in 100 on the top side only



(b) Square Sunk key :- The only difference between a rectangular sunk key and a square sunk key is that its width and thickness are equal i.e

$$W = t = d/4$$

(c) Parallel sunk key
(d) Gib-head key
(e) Feather key
(f) Woodruff key :

Effect of keyways:

- The key way cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the ~~corners~~ corners of the keyway and reduction in cross-sectional area of the shaft. In other words, the torsional strength of the shaft is reduced.

- The following relation for the weakening effect of the key way is based on the experimental results by H.F moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\,(h/d)$$

where,

e = shaft strength factor

w = width of key

d = Diameter of shaf,

h = Depth of key way = $\frac{\text{Thickness of key } (t)}{2}$

· Failure criterion of key:

(i) Shearing
(ii) Crushing

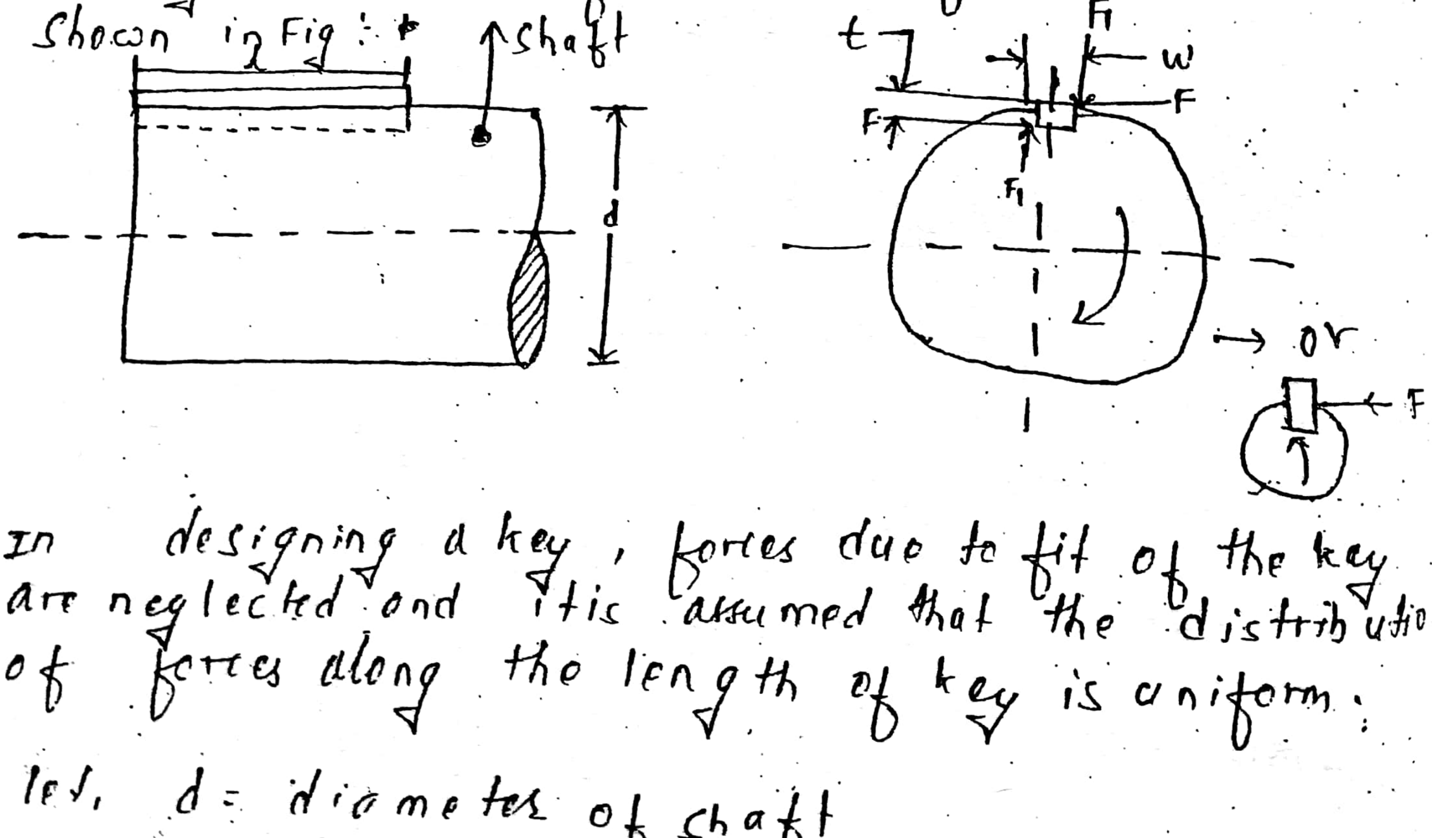
Square key → $w = t = d/4$
rect. " $w = d/4$, $t = d/6$

material - Mild steel
key seat - Groove on shaft
keyway - Groove on matting element.

V.V.I

Design of key : (Sunk key) → Design procedure.

The force acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig:-

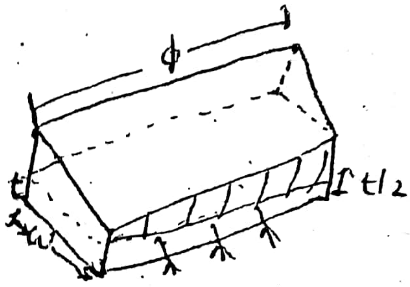


In designing a key, forces due to fit of the key are neglected and it is assumed that the distribution of forces along the length of key is uniform.

let, d = diameter of shaft
F = tangential force, acting at the circumference of the shaft.

f_s = Shear stress of the key material
f_c = crushing stress " " "
l = Length of key

$$\boxed{\begin{matrix} f_c \to \sigma_c \\ f_s \to \tau \end{matrix}}$$



For shearing.
Shearing area = lw
Tangential force, $F = f_s \cdot lw$.

Torque transmitted by key or key strength
$= F \cdot d/2$

$$\boxed{\text{key shearing strength} = f_s \cdot lw \; d/2} \quad \text{— (i)}$$

For crushing,
Crushing area $= l \cdot t/2$
$F = f_c \cdot l \cdot t/2$

$$\boxed{\text{key crushing strength} = f_c \cdot l \cdot t/2 \cdot d/2} \quad \text{— (ii)}$$

The key is equally strong in shearing and crushing if,
Shearing strength = crushing strength
$f_s \cdot lw \cdot d/2 = f_c \cdot l \cdot t/2 \cdot d/2$

or, $$\boxed{\frac{w}{t} = \frac{f_c}{2 f_s}} \quad \text{— (iii)}$$

The permissible crushing stress for usual key material is at least twice the permissible shearing stress. We know, $\omega = t$ (for square key.)

From eq (iii) when, $\omega = t$

$$\boxed{F_c = 2f_s}$$

In other words, a square key is equally strong in shearing and crushing.

→ Length of key:

Torque transmitted by ~~the~~ shaft = Torque transmitted by key

or, $\pi/16 \, f_s' \cdot d^3 = F_s \cdot l \omega \cdot d/2$

$$l = \pi/8 \cdot \frac{f_s'}{f_s} \cdot d^2/\omega$$

$$= \pi/2 \cdot d$$

$$\boxed{l = 1.571 d}$$

$F_s' = F_s$

* If shaft and key have same materia[l]

l $\omega = d/4$

Numerical example: See (Page no: 477) 13.1, 13.2, 13.3

Shaft Coupling :

shaft are usually available upto 7 meters length due to inconvenience in transport. In order to have a greater length, it becomes necessary to join two or more pieces of the shaft by means of a coupling.

The main function of coupling are :

(i) To provide the temporary connection between the shafts.
(ii) To provide misalignments between shaft
(iii) To reduce the transmission of mechanical vibration
(iv) To introduce protection against overloads.

→ Requirement of a good shaft Coupling

(1) It should be easy to connect or disconnect
(2) It should transmit the full power from one shaft to the other shaft without losses.
(3) It should hold the shafts in perfect alignment.
(4) It should to have no projecting parts

Types of shafts Couplings:

on the basis of shafts to be conn-
ed

(1) Rigid coupling: It is used to connect two shafts which are perfectly aligned.

V.V.I Types:

(a) Sleeve or muff coupling
(b) clamp or compressive coupling
(c) Flange coupling

(2) Flexible coupling: It is used to connect two shafts having both lateral and angular misalignment.

Types

(a) ~~Bu~~ Bushed pin type coupling
(b) Universal coupling,
(c) oldham couplings.

V.V.I

Sleeve or Muff-coupling :

→ It is the simplest types of rigid coupling, made of cast iron.

→ It consists of a hollow cylinder whose inner diameter is the same as that of the shaft.

→ It is fitted over the ends of the two shafts by means of a gib-head key.

→ The power is transmitted from one shaft to the other shaft by means of a key and a sleeve.

→ It is therefore, necessary that all the element must be strong enough to transmit the torque.

The usual proportions of a cast iron sleeve coupling are as follows:-

outer diameter of the sleeve, $D = 2d + 13$ mm

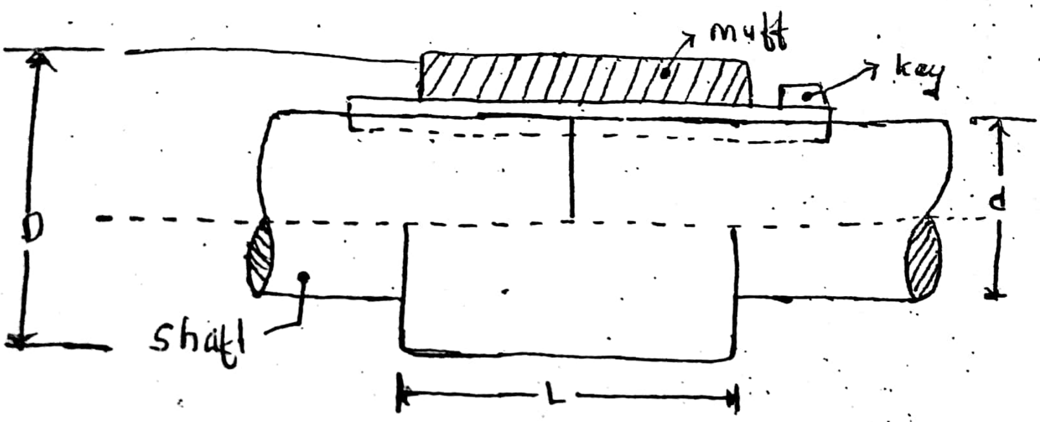
Length of the sleeve, $L = 3.5d$

where d = Diameter of the shaft.

In designing a sleeve or muff-coupling, the following procedure may be adopted.

(a) Design for sleeve :

The sleeve is designed by considering it as a hollow shaft.



Let, T = torque to be transmitted by coupling
τ_c = Permissible shear stress for the material of the sleeve which is cast iron. safe value may be taken as 14 mpa.

we know that torque transmitted by hollow section

$$T = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D} \right)$$

$$= \pi/16 \times \tau_c \times D^3(1-k^4) \quad [k = d/D]$$

(2) Design for keys:

The Length of the coupling key is at least equal to the length of the sleeve (i.e 3.5d, The coupling key is usually made into two parts so that the Length of the key in each shaft,

$$l = L/2 = \frac{3.5d}{2}$$

After fixing the length of key in each shaft, the induced shearing and crushing stresses may be checked. We know that, torque transmitted,

$T = l \times w \times \tau \times d/2$ (Considering shearing of key)

$T = l \times t/2 \times \sigma_c \times d/2$ (" crushing of ")

See (Numerical page no:- 481) example: 13·4 and 13·5 and other numerical of other copy (see most) and exercise - page no 506 Q:N - 3 and (4)

Unit. 5 (Journal Bearing)

⇒ Bearing : A bearing is a machine element which supports another moving machine element (known as journal).

It permits a relative motion between the contact surface of the members, while carrying the load.

→ Journal Bearing :

The sliding contact bearing in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as journal or sleeve bearing.

Types of bearing :

(1) Depending upon the direction of load to be supported. : The bearing under this group are :

(a) Radial bearing ↳ the load act ⊥ to dir. of motion of moving element

(b) Thrust bearing ↳ the load acts along axis of rotation.

(2) Depending upon the nature of contact.

(a) Slinding contact bearing :- the sliding take place along the surface of contact between the moving element and the fixed element. It also called plain bearing

(b) rolling contact bearing :- the steel balls or rollers are interposed between the moving and fixed elements.

V.V.I

Hydrodynamic Theory of Lubrication:

→ In hydrodynamic lubricated bearing, there is a thick film of lubricant between the journal and the bearing.

→ When the bearing is supplied with sufficient lubricant, a pressure is build up in the clearance space when the journal is rotating about an axis that is eccentric with the bearing axis

→ Bearing in which the working surfaces are completely separated from each other by the lubricant are called hydrodynamic lubricated bearings.

→ The load supporting pressure in hydrodynamic bearing aries from either:

(a) the flow of a viscous fluid in a converging channel (known as wedge film lubrication),

(b) the resistance of a viscous fluid to being squeezed out from between approaching surface (known as squeeze film lubrication).

→ The following are assumption used in the theory of hydrodynamic lubricated bearings

(1) The lubricant obeys Newton's law of viscous flow.

(2) The pressure is assumed to be constant throughout the film thickness.

(3) The lubricant is assumed to be incompressible.

(4) The viscosity is assumed to be constant throughout the film.

(5) The flow is one dimensional the side leakage is neglected.

Term Used in Hydrodynamic Journal Bearing

A hydrodynamic journal bearing is shown in figure below in which O is the centre of the journal and O' is the centre of the bearing.

D = Diameter of the bearing fig See (Page 973)

d = Diameter of the journal

l = Length of the bearing

(a) Diametral clearance : It the difference between the diameter of the bearing and the journal. mathematically diametral clearance,

$$c = D - d$$

(b) Radial clearance : It is the difference between the radii of the bearing and journal.

$$c_1 = R - r = \frac{D-d}{2} = c/2$$

(c) Diametral clearance ratio = It is ratio of the diametral clearance to the diameter of the journal. mathematically, diametral clearance ratio,

$$= \frac{c}{d} = \frac{D-d}{d}$$

(d) Eccentricity :- It is the radial distance between the centre (O) of the bearing and the displaced centre (O') of the bearing under load.

(e) eccentricity ratio :- It is ratio of the eccentricity to the radial clearance.

$$\epsilon = \frac{e}{C_1} = \frac{C_1 - h_0}{C_1} = 1 - \frac{h_0}{C_1} = 1 - \frac{2h_0}{c}$$

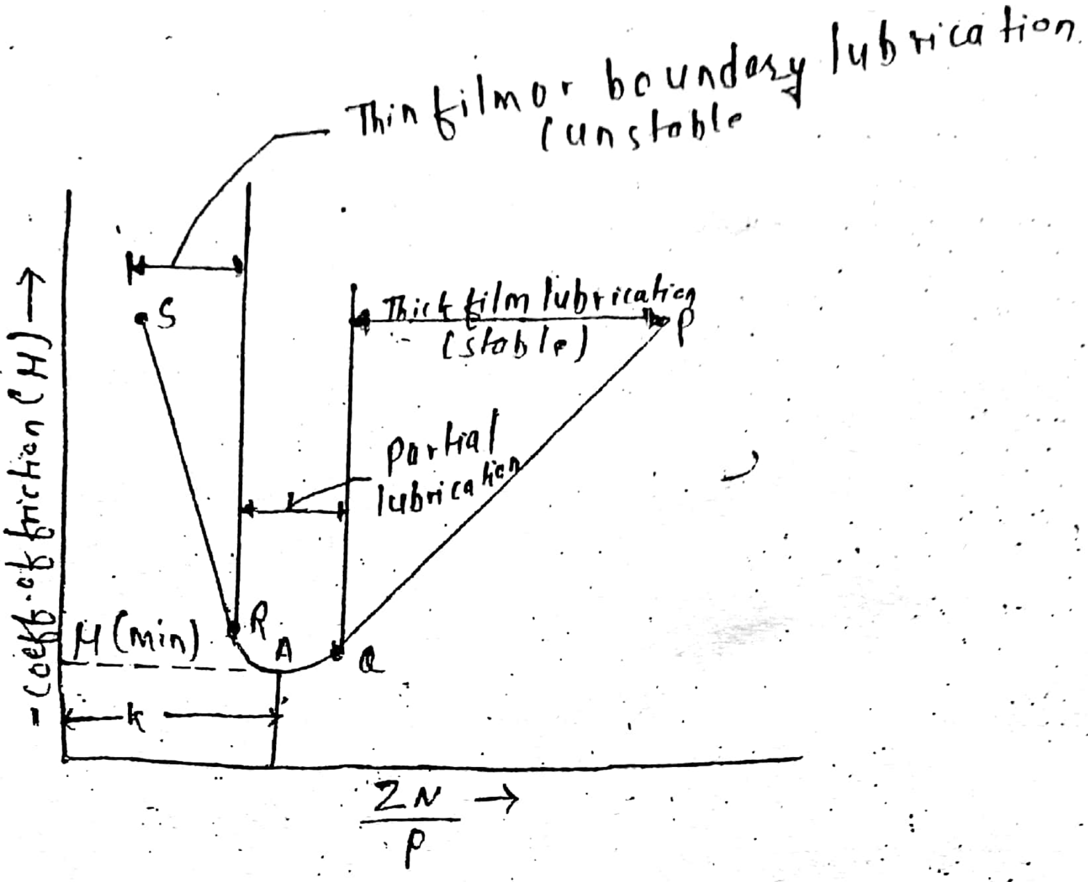
Bearing Characteristics Number and Bearing Modulus for Journal Bearings :

→ It has been shown by experiments that the co-efficient of friction for a full lubricated journal bearing is a function of three variables i.e (i) $\frac{ZN}{p}$ (ii) $\frac{d}{c}$ (iii) $\frac{l}{d}$

Therefore the coefficient of friction may be expressed as

$$\mu = \phi\left(\frac{ZN}{p}, \frac{d}{c}, l/d\right)$$

- The factor ZN/p is teasmed as bearing charasteristics number and is a dimensionless number.
- The factor ZN/p helps to predict the performance of a bearing.



→ From fig. we see that the minimum amount of friction occurs at A and at this point the value of ZN/p is known as bearing modulus which is denoted by k.

→ From above it is concluded that when the value of ZN/p is greater than k the bearing will operate with thick film Lubrication or under hydrodynamic conditions.

→ on the other hand, when value of ZN/p is less than k, then the oil film will rupture and there is a metal to metal contact.

Co-efficient of friction for journal Bearings

In order to determine the coefficient of friction for well lubricated full journal bearings, the following empirical relation established by McKee based on the experimental data, may be used.

Co-eff. of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k$$

k = factor to correct for end leakage. It depends upon the ratio of length to the diameter of the bearing (l/d)

$k = 0.0002$ for l/d ratio of 0.75 to 2.8.

Critical pressure or operating pressure:

The pressure at which the oil film break down so that metal to metal contact begins.

$$p = \frac{ZN}{4.75 \times 10^6}\left(\frac{d}{c}\right)^2\left(\frac{l}{d+l}\right)$$

Heat Generated in a Journal Bearing:

The heat generated in bearing is due to fluid friction and friction of the parts having relative motion. mathematically, heat generated in a bearing

$$Q_g = \mu \cdot W \cdot V \quad N\text{-}m/s \text{ or } J/s \text{ or watt} \quad \text{—(i)}$$

where

μ = Co-eff. of friction
W = Load on bearing in N = $P(l \times d)$
V = Rubbing velocity = $\frac{\pi d \cdot N}{60}$
N = speed in r.p.m

After the thermal equilibrium.
heat dissipated by the bearing.

$$Q_d = CA(t_b - t_a) \text{ watt} \quad \text{—(ii)}$$

V.V.I

Design procedure for Journal Bearing

When the load on journal, shaft diameter and speed are given then the following procedure may be followed for journal design:

1. ~~Design~~ Determine the length of bearing by choosing suitable value l/d from table.
2. Check the bearing pressure by comparing the calculated pressure with the pressure from table, $P = \frac{F}{ld}$

(3) Select suitable oil and its operating temperature. Find its corresponding viscosity

(4) Check $\frac{ZN}{p}$ by comparing the calculated $\frac{ZN}{p}$ with the $\frac{ZN}{p}$ from table to ensure (maintain) the hydrodynamic region.

(5) Determine the co-eff of friction.

(6) Determine heat generated at bearing surface.

(7) Determine heat dissipated at bearing surface.

(8) Check the thermal equilibrium ~~condition~~ condition to maintain heat generation is less than heat dissipation.

:- See (Numerical) :- page no: ~~26~~ 979

Ex:- 26.1, 26.2, 26.3 26.4, 26.5, 26.6 and

other old copy question

unit-6

Ball and Roller Bearing

· Introduction:

In rolling contact bearing, the contact between the bearing surface is rolling instead of sliding as in sliding contact bearing. It has a low starting friction, due to this friction offered by rolling contact bearings these are called antifriction bearing.

→ Types of Rolling contact Bearing:

There are two types of rolling contact bearing

(a) Ball bearing and (b) Roller bearing

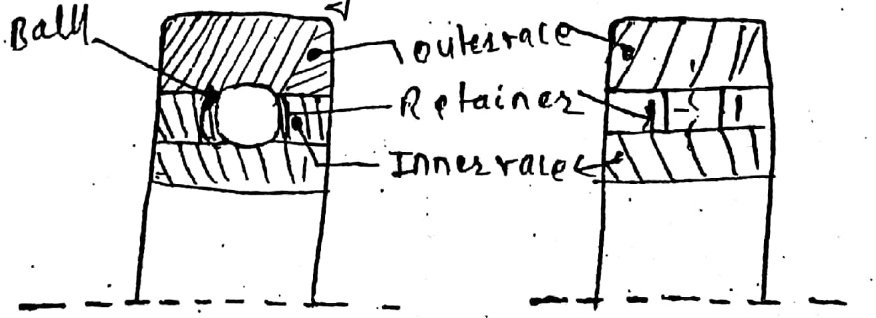


fig (a) Ball bearing (b) Roller bearing

The ball and roller bearing consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by housing or casing. In between the inner and outer race, there are balls or rollers are shown in fig. i.e ball-bearing

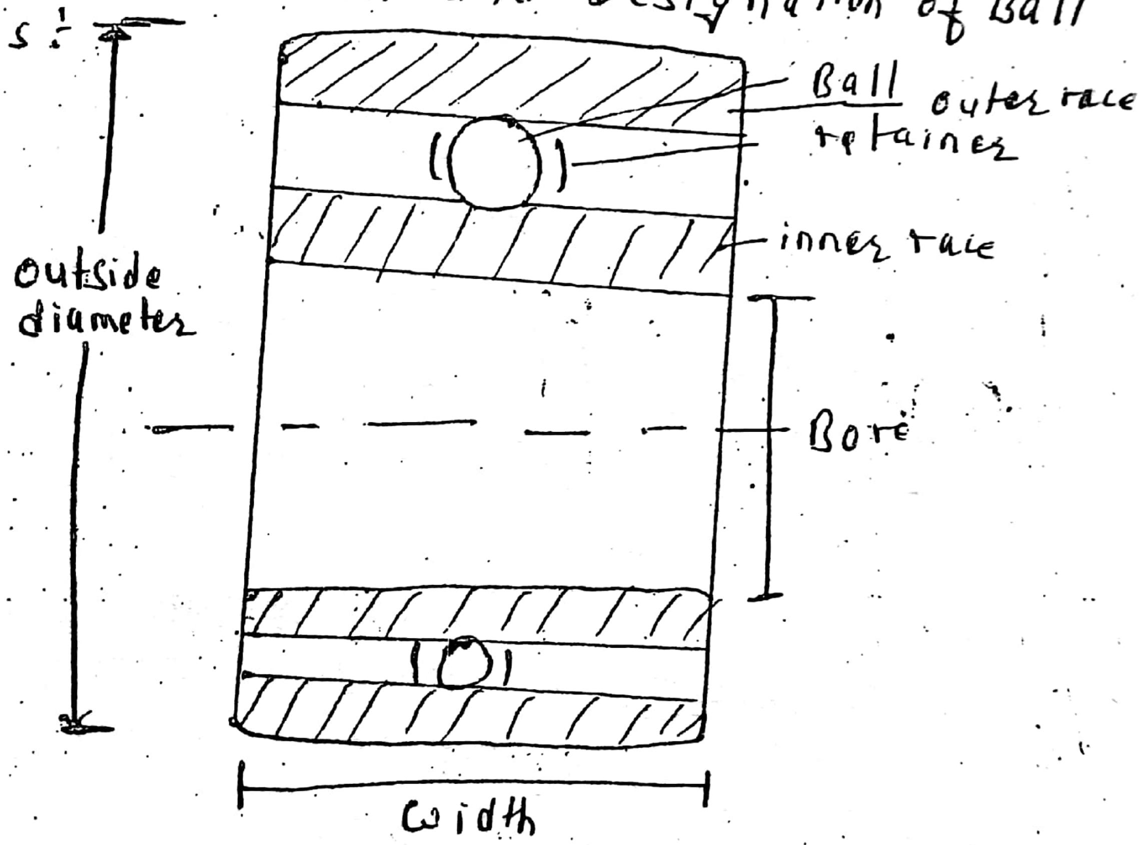
The rolling contact bearing, depending upon the load to be carried are:

(a) Radial bearing
→ Single row deep groove bearing
→ Filling notch
→ Angular contact "
→ Self aligning "

(b) Thrust bearing

→ Types of Roller bearing:
(a) cylindrical roller bearing
(B) spherical roller bearing
(C) Needle roller ''
(d) Tapered roller bearing

→ Standard Dimension and Designation of Ball Bearings:



→ The standard dimensions are given in millimeter.
→ There is no standard for the size and number of steel balls.
→ The bearing are designated by a number. In general, the number consist of atleast three digits. Addition digits or letter are used to indicate special features e.g deep groove

filling notch etc. The most common ball bearing are available in four series
(a) Extra light (100) (b) Light (200) (c) medium (300)
(d) Heavy (400)

Basic dynamic load capacity :-

Basic dynamic load capacity of bearing is the load at which bearing will attain 10^6 revolution. It is for 90% relability. It is represented by C_{10} or C.

C_{10} = basic load for 10% failure acceptable

→ Life of bearing :- The life of bearing is the no of revolution or in hours, runs the bearing before any sign of failure appear on the bearing element.

$$L = \left(\frac{C}{P}\right)^a \text{ in milions revolution}$$

L = nominal or rated life of bearing
C = basic dynamic load capacity
P = desired load
a = constant = 3 for ball bearing
$= \frac{1}{3}$ for " "

→ Rating Life (L) :- It is the life that 90% of a group of identical bearing will complete or exceed before the first evidence of fatigue develops.

→ The ~~equiva~~ equivalent load (hypothetical load)

$$P = X F_r + Y F_a$$

F_r = Constant radial load

F_a = Constant axial load

X = Radial load factor

Y = Axial load factor.

So the equivalent load is the hypothetical load which has the same effect on life of bearing co as the actual load.

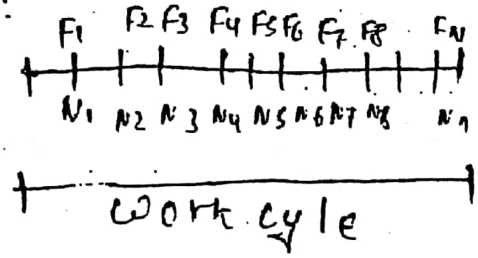
Considering race rotation factor and service factor

$$P = S(VXF_r + YF_a)$$

where, S = Service factor

V = Race rotation factor

→ <u>For variable load</u> :-



In case of variable load imposed on bearing, the work cycle is divided into small no of

portions in each of duration the variables parameters speed and load are assumed as constant,

The mean constant load or cubic mean load is given by;

$$F_m = \sqrt[3]{\frac{F_1^3 N_1 + F_2^3 N_2 + F_3^3 N_3 + \cdots}{N}}$$

where,

F_1 = constant load for N_1 revolutions
F_2 = constant load for N_2 "
$N = N_1 + N_2 + \cdots$
= total no. of revolutions
and where F_m is equivalent load.

If Speed (N) is constant.
we can express F_m in term of time instead of speed.

$$F_m = \sqrt[3]{\frac{\int_0^t F\,dt}{T}}$$

where T = time for one complete variation of load

F = load at any instant of time.

Reliability of a Bearing (R)

. It is defined as the ratio of number of bearing which have successfully completed (L) million revolution to the total number of bearing under test.

Relation between Bearing life and Reliability:

According to weibull it is given as.

$$\log_e\left(\frac{1}{R}\right) = \left(\frac{L}{a}\right)^b$$

or
$$\boxed{\frac{L}{a} = \log_e\left(\frac{1}{R}\right)^{1/b}} \quad \text{— (i)}$$

Where, L is the life of the bearing corresponding to the desired reliability R and a and b are constant whose values are.

$a = 6.84$ and $b = 1.17$

If L_{90} as the Life of bearing corresponding to a reliabity of 90% (i.e R_{90}) then

$$\frac{L_{90}}{a} = \left[\log_e\left(\frac{1}{R_{90}}\right)\right]^{1/b} \quad \text{— (ii)}$$

Dividing eq (I) by (II)

$$\frac{L}{L_{90}} = \left[\frac{\log_e(1/R)}{\log_e(1/R_{90})}\right]^{1/b}$$

$$= 6.85\,(\log_e(1/R))^{1/1.17}$$

This expression is used for selecting the bearing when the reliability is other than 90%

Relation between Bearing life and Bearing load

The approximate rating life of ball bearing is based on the fundamental eq

$$L = \left(\frac{C}{W}\right)^k \times 10^6 \text{ rev.}$$

$$C = W\left(\frac{L}{10^6}\right)^{1/k}$$

Where,

L = Rating life / Bearing life
C = Basic dynamic load rating
W = equivalent dynamic load / Bearing load
k = 3, for ball bearing
= 10/3 " roller "

→ Lubrication of ball and Roller Bearing :-

The ball and roller bearing are lubricated for following purpose.

1. To reduce friction and wear between the sliding parts of the bearing.
2. To prevent rusting or corrosion of the bearing surface.
3. To dissipate heat.
4. To ~~propet the b~~ protect the bearing surface from water, dirt etc.

Numerical :- page no :- 1011

Example :- 27.1, 27.2, 27.3, 27.4

i) According to the type of gearing:
(a) External gearing (b) Internal gearing (c) Rack and pinion.

ii) According to the position of teeth on the gear surface.
(a) straight (b) Inclined (c) curved.

Condition for constant velocity Ratio of Gears (law of Gearing)

Fig: See page no: (1027)

Consider the portions of the two teeth, one on the wheel 1 and other on wheel 2.
Let the two teeth comes in contact at point Q, and the wheels rotate in the direction.

If the teeth are to remain in contact, then the components of these velocities along the common normal MN must be equal

$\therefore \; V_1 \cos\alpha = V_2 \cos\beta$

or, $(\omega_1 \times O_1Q) \cos\alpha = (\omega_2 \times O_2Q) \cos\beta$

or, $(\omega_1 \times O_1Q) \cdot \dfrac{O_1M}{O_1Q} = (\omega_2 \times O_2Q) \dfrac{O_2N}{O_2Q}$

or, $\omega_1 \cdot O_1M = \omega_2 \cdot O_2N$ ——— (i)

or, $\dfrac{\omega_1}{\omega_2} = \dfrac{O_2N}{O_1M}$

Also, from similar triangles O_1MP and O_2NP

$$\frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \quad \text{—(ii)}$$

Combining eq (i) and (ii) we get;

$$\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M} = \frac{O_2P}{O_1P} \quad \text{—(iii)}$$

From eq (iii), we see that the angular velocity ratio is inversely proportional to the ratio of distance of P from the centres O_1 and O_2 or the common normal to the two surfaces at the point of contact & intersects the line of centres at point P which divides the centre distance inversely as the ratio of angular velocities.

- Therefore, in order to have a constant angular velocity ratio for all positions of the wheels, and must be the fixed point (called pitch point) for the two wheels.
- In other words, the common normal at the point of contact between a pair of teeth must always pass through the pitch point.
- This is fundamental condition which must be satisfied while designing the profiles for the teeth of gear wheels.

It is also known as law of gearing.

unit-7. (Gears)

Introduction :
Gear is a machine element which is used to transmit power from one point to another.

→ Spur Gears :
These gears have teeth parallel to the axis of the wheel

⇒ Advantage of gear drives :
(a) It transmit exact velocity ratio.
(b) It may be used to transmit large powers.
(c) It has high efficiency.
(d) It has reliable service.
(e) It has a compact layout.

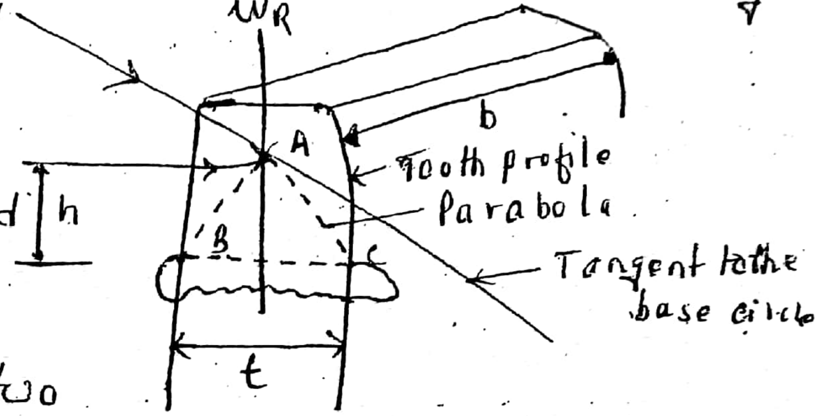
⇒ Classification of Gears :
The gear or toothed wheels may be classified as follow :
(1) According to the position of axes of the shafts.
(a) Parallel (b) Co planner

The two paralel and co-planner shaft connected by gear are called spur gear
→ helix gear

(2) According to the peripheral velocity of the gear
(a) Low velocity (b) medium velocity (c) High velocity

Beam Strength or Gear teeth - Lewis equation

Lewis assumed that as the Load is being transmitted from one gear to another, it is all given and taken by one tooth, because it is not always Safe to assume that the Load is distributed among Several teeth.



consider each tooth as a cantilever beam loaded by a normal load (W_N)

W_T

h

- It is resolved into two components i.e tangential components ie (W_T) and radial component (W_R) acting perp. and parallel to the centre line of tooth respectively.

Fig:- Tooth of a gear.

- The tangential component (W_T) induces bending stress which tends to break the tooth.

- we therefore concluded that the section BC is the section of max^m^ stress or the critical Section.

The max^m^ value of the bending stress at section BC is given by

$$\boxed{\sigma_w = \frac{M \cdot y}{I}} \quad \text{——— (i)}$$

Where, M = Max^m bending moment at the critical section BC
$= W_T \times h$

W_T = Tangential load acting at the tooth

h = Length of the tooth

y = Half the thickness sect. BC $= t/2$

I = MOI $= \frac{bt^3}{12}$

b = width

Substituting the value of M, y and I in eq (i)

$$6\omega = \frac{(W_T \times h) \times t/2}{bt^3/12} = \frac{W_T \times h \times 6}{bt^2}$$

$$W_T = \frac{6\omega \times b \times t^2}{6h}$$

let, $t = n \times p_c$
$h = k \times p_c$

where, n and k are constant

$$\therefore W_T = \frac{6\omega \times b \times (n \times p_c)^2}{6 \times (k \times p_c)}$$

$$= \frac{6\omega \times b \times n^2 \cdot p_c}{6 \times k \cdot p_c}$$

$$\boxed{W_T = 6\omega \cdot b \cdot \pi m \cdot y}$$

$y = \frac{n^2}{6k}$

$\therefore p_c = \pi m$

$\Downarrow$ circular pitch

$m = \frac{D}{T}$

W_T = Beam strength of the tooth

y = Lewis form factor or tooth form factor

~~The quantity~~

Note :-

$$y = \frac{x^2}{6t} = \frac{t^2}{p_c^2} \times \frac{p_c}{6h} = \frac{t^2}{6h \cdot p_c}$$

therefore in order to find the value of y, the quantities t, h and pc may be determined

→ The value of y is independent of the size of the teeth and depends only on the no of teeth on a gear and the system of teeth.

→ The value of y in terms of no. of teeth may be expressed as follows.

$y = 0.124 - \frac{0.684}{T}$, for 14.5° composite and full depth involute system

$y = 0.154 - \frac{0.912}{T}$, for 20° full depth " "

$y = 0.175 - \frac{0.841}{T}$ for 20° stub system.

Causes of Gear Tooth Failure :

(1) Bending Failure i.e $W_D > W_S$ ⇒ gear tooth will Fail in bending

(2) Pitting i.e surface contact stress > endurance limit of material

(3) Scoring i.e Excessive surface pressure → excessive heat generation.

(4) Abrasive wear i.e foreign particles in lubricants such as dirt / dust.

(5) Corrosive wear :-

Corrosion of tooth surface is caused due to the presence of corrosive element such as additives present in lubricating oils.

Angle of obliquity (Pressure angle)

→ It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point.

→ It is usually denoted by ϕ

→ The standard pressure angles are $14\frac{1}{2}°$ and $20°$

V.V.I

Design Procedure for Spur Gears:

In order to design spur gears, the following procedure may be followed.

(1) First of all, the design tangential tooth load is obtained from the power transmitted and the pitch line velocity by using the following relation:

$$\boxed{W_T = \frac{P}{V} \times C_s} \quad \text{— (1)}$$

where, W_T = permissible tangential tooth load in N

P = Power transmitted in watts

V = Pitch line velocity. $= \frac{\pi D N}{60}$ m/s

D = Pitch circle

N = rpm speed

C_s = Service factor.

(2) Apply the Lewis eqn as follows:

$$W_T = \sigma_\omega \cdot b \cdot P_c \cdot y = \sigma_\omega \cdot b \cdot \pi m \cdot y$$

$$= (\sigma_0 \cdot C_v)\, b \cdot \pi m \cdot y \quad (\sigma_\omega = \sigma_0 \cdot C_v$$

allowable static stress

(3) calculate the dynamic load (W_D) on the tooth by using Buckingham eqn

$$\boxed{\begin{aligned} W_D &= W_T + W_I \\ &= W_T + \frac{21v\,(b\cdot C + W_T)}{21v + \sqrt{b\cdot C + W_T}} \end{aligned}}$$

Dynamic factor $(C) = \dfrac{k\cdot e}{\frac{1}{E_P} + \frac{1}{E_G}}$

In calculating the dynamic load (W_D), the value of tangential load (W_T) may be calculated by neglecting the service factor (C_S) i.e

$$W_T = P/v$$

(4) Find the static tooth load (i.e beam strength or endurance strength of the tooth) by using the relation,

$$\boxed{W_S = \sigma_e\cdot b\cdot P_c\cdot y = \sigma_e\cdot b\cdot \pi m\cdot y}$$

For safety against breakage, W_s should be greater than W_D. i.e $(W_S > W_D)$

(5) Finally, find the wear tooth load by using the relation,

$$\boxed{W_w = D_P\cdot b\cdot Q\cdot K}$$

where,

The wear load (W_w) should not be less than the dynamic load (W_D) i.e $(W_w$

~~Q~~ Q

D_p = pitch circle diameter of pinion pn

b = Face width of pinion is mm

k = Load stress factor = in N/mm²

Q = Ratio factor = $\frac{2 \times V.R}{V.R+1} = \frac{2T_G}{T_G+T_p}$

$$Q = \frac{2 \times V.R}{V.R - 1} = \frac{2T_G}{T_G - T_p}$$, for internal gears

$V.R.$ = Velocity ratio = $\frac{T_G}{T_p}$

$$k = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left(\frac{1}{E_p} + \frac{1}{E_G}\right)$$

where,

σ_{es} = surface endurance limit in N/mm²

ϕ = Pressure angle

E_p = Young's modulus for material of pinion in N/mm²

E_G " " " gear in N/mm²

Numerical. Pg-1047

Example: 28.1, 28.2 28.3, 28.4, 28.5, 28.6, 28.7

key

Design a square ~~key~~ key for fixing a gear on the shaft which transmit 10kw power at 720rpm. The shaft and key are both made of plane carbon steel (C45) and factor of safety is 3.

Pageno - 476

Key - Numerical

Example : (13.1)

$d = 50mm$

$\tau = 42 mpa$

$= ~~0~~.42 N/mm^2$

$\sigma_c = 70 N/mm^2$

For a ~~given~~ Design rectangular key

$$\boxed{\begin{aligned} \omega &= d/4 \\ &= 50/4 = 12.5 \end{aligned}}$$

From table

$\omega = 16 mm$ Ans

$t = 10mm$ Ans

Let, l be the Length,

$T = l \times \omega \times \tau_s \times d/2$

$= l \times 16 \times 42 \times 50/2 = 16800 l$ N-m

Now, we know,

Torsional / shearing stress $= \frac{\pi}{16} \tau_s \times d^3$

$= \pi/16 \times 42 \times (50)^3$

$= 1.03 \times 10^6$ N-mm

From above formula

$$d = \frac{1.03 \times 10^6}{16800} = 61.31 \text{ mm}$$

Now,

For chrushing key,

$$T = d \times \sigma_c \times t/2 \times d/2$$

$$= d \times 70 \times 10/2 \times 50/2 = 8750 d \text{ N-mm}$$

$$d = \frac{1.03 \times 10^6}{8750} = 117.7 \text{ mm}$$

Taking largest of the two values

$d = 117.7$ say $\underline{120 \text{ mm}}$